The existence of low-end firms may help high-end firms

September 20, 2007

The second-round version for Marketing Science

Abstract

In this paper, two examples of competition between high-end and low-end products benefiting the high-end firms are presented. One is a quantity competition model, and the other is a price competition model with product differentiation. The key factor is the existence of two heterogeneous consumer groups: those who demand only high-end (name-brand) products and those who care little whether products are high- or low-end. We show that, under certain conditions, the profits of firms in the high-end market are larger when there are firms producing low-end products than when there are not. The result provides a new theoretical mechanism concerning the profitability and pricing of national brand firms after the entry of private labels. It also has several implications for pricing and marketing strategies. For instance, established firms should not decrease their prices after the entry of nonestablished firms. Established firms should more earnestly persuade customers that their products are branded goods after the entry of nonestablished firms.

Key words: marketing strategy, pricing research, product positioning, game theory

1 Introduction

In this paper, two examples of competition between high-end and low-end products benefiting the high-end firms are presented. One is a quantity competition model, and the other is a price competition model with product differentiation. The key factor in our examples is the existence of two heterogeneous consumer groups. One consists of consumers who demand high-end (name-brand) products. Low-end (private-label) products are worth little to them. The other consists of consumers who care little whether products are high- or low-end. Therefore, they buy products with the lowest price.[1] Based on these heterogeneous consumer groups, we show that, under certain conditions, the profits of firms in the high-end market become larger when there are firms in the low-end market than when there are not.[2]

The logic behind our result is as follows. If no firm is in the low-end market or if the prices of low-end products are sufficiently high, the high-end firms have incentives to sell their products to low-end consumers. Of course, once the high-end firms sell their products to them, the prices in the high-end market collapse. If the increase in the sales volume is offset by the decrease in price, the existence of

[1] Computer markets may be a good example of such heterogeneous groups of users. Computers designed for home use usually perform better than those designed for business use. This is because home users use PCs for various purposes: writing documents, listening to music, editing pictures, and watching movies. Computers that perform poorly in image processing are of no use to most home users. However, such computers are adequate for business users who only write documents and browse the Internet.

[2] Rosenthal (1980) adopts a similar setting to analyze the relationship between price dispersion and the number of suppliers. He considers two classes of consumers: those who view labels of companies as artifacts and purchase only from low-price companies, and those who perceive significant differences among brands and purchase only their respective favorite brands (see Rosenthal (1980, p.1575)). He shows that the equilibrium price increases as the number of firms increases. In his model, however, pure-strategy equilibria do not exist, and the increment of the equilibrium price is evaluated on the concept of stochastic dominance. In Rosenthal (1980) and most of the subsequent studies (e.g., Narasimhan (1988) and Baye *et al.* (2004)), there are discussions of price dispersion, but the authors do not consider the relationship between the profitability of incumbent firms and the existence of low-end firms.

the low-end market decreases the profits of the high-end firms. A sufficient supply of low-end products makes the low-end market unprofitable for high-end firms and removes the high-end firms' incentives to produce more. Therefore, rivals in the low-end market become beneficial to high-end firms. Note that the existence of low-end firms raises the price and decreases the supply of high-end products.

Our results have an implication for pricing strategies. The optimal pricing for high-end products need not be monotonically decreasing in the degree of competition in the corresponding low-end market. If the market structure is as stated above, high-end firms should set high prices *because of* severe competition in the low-end market.[3]

Given the main result of our paper, firms that produce high-end products should persuade customers that their products are high-end ("premium") goods. Therefore, activities that enhance the brand equity of firms might be much more important in markets, as stated above. We now discuss several ways to conduct such activities.

First, if brand values matter in a market, the owner firm should place more reliance on advertising to create the perception of its brand as "premium" by high-end consumers. Once the brand is perceived as such, the firm can earn additional profit by competing against firms in the corresponding low-end market.

Second, we believe that the result in Randall *et al.* (1998) implies another route for such promotional activities. They show that the presence of "premium" or high-quality products in a product line enhances brand equity.[4] Based on their research, if a "high-end" brand (brand equity) in a market is associated with other high-quality products in its product line, to protect its profit in the market, it is beneficial for

[3] Hauser and Shugan (1983) is a pioneering work about the relation between competitiveness and marketing strategies. Recently, along the same lines, Sayman *et al.* (2002) and Steenkamp *et al.* (2005) discuss these matters using empirical data.

[4] Keller and Lehmann (2006) provide an excellent survey and direction for future research concerning brands and brand equity.

a firm to enhance the brand equity of its *other* high-quality products. Promotional activity may indirectly enhance the profitability of a product in the market.[5]

Furthermore, if technological progress plays an important role in creating a high-end product, firms should spend more money on drastic innovations. The reason is the same: once the product satisfies the technological requirements that high-end consumers demand, a firm can ensure that its profit is stable regardless of how competitive the low-end market becomes.

There is marketing literature that seems to be consistent with our arguments. First, as summarized in Soberman and Parker (2004), some empirical studies show the existence of heterogeneity of consumer preferences for national brands and private labels (or generic brands): some consumers are willing to pay more for advertised (name-brand) products, whereas others believe that private-label products are the same as name brands in regard to overall quality, taste, availability, freshness, guarantee of satisfaction, clarity of labeling, and quality of packaging, among other attributes.

Second, Pauwels and Srinivasan (2004) empirically show that the invasion of private-label food products increases the *profits* from name-brand (premium brand) goods if consumers regard the quality of the name brand as being much higher than that of a private label. Although the fundamental structure of the food product industry is not exactly the same as our setting, our logic might apply, with a slight

[5] When the firm employs these promotional activities, it has to take into account the caveat of Leclerc *et al.* (2005). They have shown that, in separate evaluations, people are predisposed to use firm information (how the item ranks *within the firm*) as a frame of reference to evaluate the quality of that item. As a result, customers may evaluate a high-quality item from a low-ranked firm as being better than a low-quality item from a high-ranked firm. To overcome this propensity, firms have to induce customers to focus their attention on the differences *between the firms*.

modification.[6] [7]

Now, we discuss the related theoretical literature. To the best of our knowledge, no previous study has shown that the *profits* of high-end firms *increase* through competition with low-end firms.[8] However, there are two papers with results that are closely related to ours. One is by Coughlan and Soberman (2005), who consider the manufacturers' distribution problem, i.e., whether or not they establish their own outlet stores given independent primary retailers. One of their results is that outlet stores might benefit independent retailers. The other is Chen and Riordan (2007), who construct a kind of monopolistic competition model with horizontal differentiation. They show that, under certain conditions, an additional new entry increases existing firms' profits. There are two clear differences between these two studies and ours. First, our motivation is quite different from theirs. As stated at the beginning of this section, we are interested in the relationship between the profits and competition in the context of the vertical (*i.e.,* high- and low-end) structure of

[6] Pauwels and Srinivasan (2004) provide a plausible explanation for their finding that premium brands do not directly compete with private labels, but they instead focus on serving core brand-conscious consumer segments with the introduction of new product varieties. Our logic might be a theoretical explanation of their interpretation.

[7] Unfortunately, we could not find any other empirical papers that investigate the relationship between entries and incumbent firms' profits. We think that one of the major difficulties lies in obtaining data; researchers often cannot access a firms' profit data and/or it is difficult to extract the exact effects of entries from the complicated profit data. As indirect empirical support for our analysis, some studies have reported that high-end prices increase as the degree of competition in the low-end market increases. (Recall that, in our argument, the existence of low-end firms raises the price of high-end products and decreases the supply of high-end products.) Ward *et al.* (2002) show empirically that increases in the share of private-label goods are correlated with a rise in the prices of name-brand goods. Frank and Salkever (1997) provide evidence from the pharmaceutical industry that brand-name drug prices increase after the entry of generic drugs into the market and are accompanied by large decreases in the prices of generic drugs in general.

[8] In the context of market entry, however, there is some literature in which it is argued that a new entry increases the *price* of an incumbent firm's product. Inderst (2002) considers how prices react to an increase in competition. Davis *et al.* (2004) show that a low-end firm's entry makes the incumbent high-end firm's price higher than the monopoly price. See also Satterthwaite (1979), Stiglitz (1987), and Schultz and Stahl (1996).

products and consumers. Second, we show the results not only in price competition, to which both of their models belong, but also in quantity competition.[9] Nevertheless, there are some common interesting factors between their results and ours. Therefore, we discuss this issue in greater depth in Section 3 where we investigate our price competition model.

The remainder of the paper is organized as follows. In the next section, we describe a simple Cournot duopoly game and analyze the model by considering two cases: when there is no firm in the low-end market and when there are many firms in the low-end market. Then, we derive the Cournot-Nash equilibria in each case, and we derive the main result. In Section 3, we construct a Bertrand model with product differentiation and derive a similar result in Section 2, in which entries might benefit incumbent firms. The last section is the conclusion.

2 Cournot competition

2.1 Model

We consider an industry with two differentiated products (h and l). For convenience, we call h and l high-quality and low-quality products, respectively. There are two major firms (1 and 2) that produce h at a constant marginal cost normalized to zero.[10] No fixed cost is assumed for production. In this section, we consider quantity competition. Let q_i be firm i's output level. In addition, define $q = (q_1, q_2)$.

[9] As mentioned in Cabral (2000, p.113), which is more realistic, Cournot or Bertrand, depends on what industries we consider. For instance, if capacity and output can be easily adjusted, a Bertrand model is a better approximation. Software, insurance, and banking industries can be described by Bertrand models. On the contrary, if capacity and output are difficult to adjust, a Cournot model is a better approximation. Cement, steel, automobile, and computer industries can be described by Cournot models. Therefore, we believe that it is worthwhile to provide our results in both settings.

[10] Although the two firms are the only players in our game, we also implicitly consider firms that produce l at a constant marginal cost normalized to zero. In the next section, we analyze two cases: (i) no firm in the low-end market; (ii) perfect competition in the low-end market.

We assume two groups of consumers, H (the high-end market) and L (the low-end market). For simplicity, we consider a polar case of the heterogeneity of consumer groups. The consumers in H demand only h. That is, the quality of l is not at all sufficient for the consumers in H.

[Figure 1 here]

Let p^h be the price of h. The demand function of this high-end market, $D^H(p^h)$, is given by[11]

$$D^H(p^h) = \begin{cases} 0 & \text{if } p^h \in (1, \infty), \\ 1 - p^h & \text{if } p^h \in [0, 1]. \end{cases}$$

The consumers in L are indifferent between h and l. In other words, the high quality of h (compared with l) is of no value to consumers in L. Let p^l be the price of l. The demand function of this low-end market, $D^L(p^l)$, is given by[12]

$$D^L(p^l) = \begin{cases} 0 & \text{if } p^l \in (a, \infty), \\ b(1 - p^l/a) & \text{if } p^l \in [0, a]. \end{cases}$$

We assume $0 < a \leq 1$. Note that $D^L(p^l)$ is a linear demand function such that the highest willingness to pay is given by a and the largest demand (at $p^l = 0$) is given by b. Thus, (a, b) measures the relative market properties of the low-end market taking the high-end market as a reference point.[13]

In the following subsections, we consider two polar cases: (1) no firm produces product l; (2) many firms produce product l. Comparing the two cases, we derive the main result of this study.

[11] This demand function is derived by assuming that a typical consumer in H has the willingness to pay x for product h, x is distributed uniformly on $[0, 1]$, and the total population is 1.

[12] This demand function is derived by assuming that a typical consumer in L has the same willingness to pay x for product h or for product l, x is distributed uniformly on $[0, a]$, and the total population is b.

[13] Although $a \leq 1$ is assumed for simplicity, it seems reasonable to assume that quality-conscious consumers tend to evaluate high-quality products at least as high as quality-unconscious consumers. As for b, no upper bound is assumed. Therefore, our analysis can cover various demand structures on the relationship between the high-end and the low-end markets.

2.2 Case I: No firm in the low-end market

In this subsection, we consider a case in which no firm produces l and the two major firms can potentially sell to both groups of consumers.

We describe how the price p^h is determined given the two groups of consumers. As long as $1-(q_1+q_2) \geq a$, no consumer in L buys h. Therefore, p^h is given by $1-q_1-q_2$. If $1-(q_1+q_2) < a$, some consumers in L buy h. Because h and l are completely indifferent to the consumers in L, p^h is determined so that it satisfies $D^H(p^h)+D^L(p^h) = q_1+q_2$. This means $p^h = a(1+b-(q_1+q_2))/(a+b)$.

In summary, p^h is determined as follows.

$$p^h(q) = \begin{cases} 1-q_1-q_2 & \text{if } q_1+q_2 \leq 1-a \\ \dfrac{a(1+b-(q_1+q_2))}{a+b} & \text{otherwise.} \end{cases} \tag{1}$$

Let $\pi_i(q)$ be the profit function of firm i. For $i=1,2$, this can be expressed as follows.

$$\pi_i(q) = \begin{cases} (1-q_1-q_2)q_i & \text{if } q_1+q_2 \leq 1-a, \\ \dfrac{a(1+b-(q_1+q_2))}{a+b}q_i & \text{otherwise.} \end{cases} \tag{2}$$

In this case, the game becomes a simple Cournot duopoly game with a kinked demand curve.

There are two candidates of Cournot equilibria:

$$q^x \equiv (1/3, 1/3), \quad q^y \equiv ((1+b)/3, (1+b)/3).$$

This is caused by our kinked inverse demand function. q^x (*resp.* q^y) is the Cournot equilibrium when the whole inverse demand function is given by $p^x(q) = 1-q_1-q_2$ (*resp.* $p^y(q) = a(1+b-(q_1+q_2))/(a+b)$) (see (1)). Therefore, we must check not only the local optimality (*i.e.,* first-order condition) but also the global optimality (*i.e.,* deviations "beyond" the kinked point). After several calculations, we obtain the following lemmas (the calculations are described in Appendix):

Lemma 1 $q^y = ((1+b)/3, (1+b)/3)$ *becomes a Cournot equilibrium if* $a \geq (2-b)^2/(3(4+b))$ *or* $b \geq 2$. *Each firm obtains* $a(1+b)^2/(9(a+b))$.

Lemma 2 $q^x = (1/3, 1/3)$ *becomes a Cournot equilibrium if* $a \leq 4/(3(4+3b))$ *is satisfied. Each firm obtains* $1/9$.

Lemma 1 says that q^y becomes an equilibrium if the low-end market is sufficiently profitable (*i.e.,* a is relatively large for a given b) or large (*i.e.,* b is higher than 2). The first condition is relatively easy to understand. Firms sell to the consumers in L because the increase in sales outweigh the decrease in price. What the second condition implies is slightly complicated. Although a firm must decrease its quantity drastically to satisfy $q_1 + q_2 \leq 1 - a$ (*i.e.,* raising the market price by selling only to the consumers in H), it is impossible to satisfy $q_1 + q_2 \leq 1 - a$ given $q_1 = q_2 = (1+b)/3 \geq 1$ under $b \geq 2$. Therefore, no firm has an incentive to raise the market price even if the equilibrium market price is very low.

In general, the condition in Lemma 2 says the inverse. That is, firms sell only to high-end consumers if the low-end market is sufficiently unprofitable and/or inelastic.

It is noteworthy that this game has both equilibria under a certain range of (a, b) because it is possible to satisfy both $a \geq (2-b)^2/(3(4+b))$ and $a \leq 4/(3(4+3b))$.

2.3 Case II: Many firms in the low-end market

In this subsection, we assume that there are so many minor firms competing in the low-end market that no major firm wants to sell its product to the consumers in L. For simplicity, we assume perfect competition in the low-end market and $p^l = 0$. The setting is similar to the case in which the two major firms cannot supply to the low-end market.

We can say that the two major firms play a simple Cournot duopoly game with $D^H(p^h)$. A simple calculation shows that the Cournot-Nash equilibrium is q^x and each firm's equilibrium profit is 1/9. We summarize this result as follows.

Lemma 3 *If l is sufficiently supplied by minor firms, the unique Cournot-Nash equilibrium is $q^x = (1/3, 1/3)$, and firms 1 and 2 obtain $1/9$.*

2.4 Comparison

Using the results obtained so far, we determine the condition under which case II is more profitable than case I for the two major firms.

First of all, q^y must be a Cournot equilibrium. Otherwise, q^x becomes a unique equilibrium in Case I. From Lemma 1, q^y becomes a Cournot equilibrium if one of the following inequalities is satisfied: $a \geq (2-b)^2/(3(4+b))$ or $b \geq 2$.

Second, we need $\pi_i(q^y) \geq \pi_i(q^x)$. That is, case II is more profitable than case I. This condition can be rewritten as follows:

$$\frac{1}{9} > \frac{a(1+b)^2}{9(a+b)} \quad \Leftrightarrow \quad a < \frac{1}{2+b}.$$

Therefore, we obtain the following result.

Proposition 1 *If $a < 1/(2+b)$ holds and either $a \geq (2-b)^2/(3(4+b))$ or $b \geq 2$ is satisfied, then an adequate supply of l is beneficial to the two major firms.*

Figure 2 shows the region in which this proposition holds. It is noteworthy that $a < 1/(2+b)$ does not hold for any $b > 0$ as long as $a > 1/2$.

[Figure 2 here]

At first glance, the availability of the low-end market is (weakly) beneficial to the two major firms. At least, it never seems to be harmful to them. Indeed, if the low-end market is sufficiently large in terms of both willingness to pay (measured by a) and market size (measured by b), the major firms are better off by selling their products to the consumers in L. In this case, they never want any entry in the low-end market. On the contrary, if the low-end market is highly elastic (a is too small), the major firms never sell h products to the consumers in L in order to avoid price collapse. In this case, the entries in the low-end market have nothing to do with the major firms' profitability because the l product is not a substitute of the h product for the consumers in H.

However, if we explicitly take the strategic interaction between the high-end firms into account, the entries of low-end firms increase the profits of the major firms under certain conditions. In other words, the elimination of the low-end market increases the major firms' profits. A rough intuition of this argument is as follows.

First, we should recall that $q^y = ((1+b)/3, (1+b)/3)$ is the pair of the locally stable quantities supplied. This implies that, if firm i deviates from q_i^y, it must choose its production level so that the sum of products is less than or equal to $1-a$ (see (1)). Based on this property, we show that q^y might be globally stable (i.e., a Nash equilibrium) *even when* q^x is more profitable than q^y. Roughly speaking, there are two types of reasons: (i) a type of "prisoners' dilemma" and (ii) a type of "coordination failure" (see Figure 2).

Type (i) occurs when a is relatively large for a given b. In this case, even though q^x is more profitable than q^y, q^x cannot be a Nash equilibrium.[14] At q^x, each firm has an incentive to deviate by selling to the consumers in L, who have relatively high willingness to pay. We should note that this deviation becomes profitable because the market price of h products is still high due to the small quantity supplied by the other firm (i.e., $q_j^x = 1/3$). Once q^x cannot be realized as a Nash equilibrium,

[14] If $a > 1/3 (> 4/(3(4+3b)))$, it is clear that q^x cannot be a Cournot equilibrium. In this case, some consumers in L buy h given q^x because $1 - q_1^x - q_2^x = 1/3 < a$. Therefore, q^x is no longer locally stable.

both firms increase their production level following the best response functions on $p^h(q)$ under $q_1 + q_2 > 1 - a$.

Type (ii) occurs when a is relatively small for a given b. It differs from type (i) in that q^x can be a Nash equilibrium in this case.[15] Therefore, once both firms choose q^x, no firm wants to sell its product to the consumers in L. However, if firms choose q^y initially, each firm has no incentive to decrease its production level. As indicated in the explanation of Lemma 1, given the other firm's large quantity, a firm cannot obtain a price increase that will be high enough to compensate its reduction of production.[16] Therefore, we regard this situation as a typical coordination failure.

We have to note that *multiple high-end firms are needed* to derive our main result. In other words, if there is only one high-end firm, the existence of low-end firms is always detrimental. When there is only one high-end firm, it can set its quantity of production without taking the rival (high-end) firm's response into account.[17] If the firm considers that supplying its product to consumers at H and L is optimal, then it will do so; otherwise, it will not. The existence of low-end firms deprives the monopolist of this kind of freedom. Therefore, the low-end firms do not provide any benefit to the high-end firm.

For simplicity, we assume perfect competition in the low-end market in this section. However, Proposition 1 does not depend on the number of producers of l. In the Appendix, we show that the same result is obtained even if there is only one firm in the low-end market. This guarantees that our result does not depend on the number of firms in the low-end market.[18] In other words, our result holds even

[15] See Lemma 2.

[16] If we consider this problem in the context of the long-term relationship, q^x might be reached from q^y. Even though a firm that initially decreases its production level must incur some temporary loss, the other firm would respond by decreasing its production level in the near future. Because q^x is more profitable to q^y, the initial loss would be compensated in the long run.

[17] Clearly, there is no factor such as a prisoners' dilemma or coordination failure.

[18] Of course, if our result holds for one low-end firm case, we can also obtain it for a multiple-firm

when the degree of competition in the high-end market is higher than that in the low-end market.

3 Bertrand competition

We now examine whether or not our finding holds in a price competition model. This section relies significantly on Coughlan and Soberman (2005), who consider the relationship between consumer heterogeneity and the distribution strategies of competing manufacturers, i.e., whether or not the manufacturers should use outlet stores in addition to independent primary retailers. One of their findings is that independent primary retailers might benefit from the existence of (competing) outlet stores. The basic logic underlying this result is that, if price-sensitive (low-end) and less price-sensitive consumers are segmented by the appearance of outlet stores, primary retailers can obtain more profit from the less price-sensitive consumers by maintaining high prices. Although their motivation is quite different from ours, their result is also useful for our purpose in this section. In the following, we adjust their model and interpretations to our concern. We then derive essentially the same result in our context.

3.1 Model

Suppose two differentiated products (h and l). As in Section 2, we call h and l high-quality and low-quality products, respectively. Consider a linear city along the unit interval $[0, 1]$, where firm 1 is located at 0 and firm 2 is located at 1. Those firms are major firms that produce h at a constant marginal cost normalized to zero. Therefore, we consider the horizontal differentiation between h products. Two

case because the total output level of product l increases in the number of low-end firms. It is also noteworthy that this implies that our result holds even if the existence of an entry cost restricts the number of firms in the low-end market.

types of consumers (H and L) are uniformly distributed along the interval. The two segments differ in two ways. First, they differ in their transport cost for h. The consumers in H have a (common) high transport cost (t_H), while the consumers in L have a (common) low transport cost (t_L), where $t_H > t_L$. In other words, the consumers in L are more price-sensitive and have higher price elasticity of demand than those in H. To simplify the notation, we normalize the value of t_H to be 1, and then $t_L < 1$. Second, similarly to Section 2, they differ in the willingness to pay for l. The consumers in H demand only h, and those in L are completely indifferent between h and l. The total number of consumers in the market is normalized to one, and the number of consumers in H (L) is λ $(1 - \lambda)$.

Each consumer demands, at most, one unit. If a consumer located at x buys h, her utility is given by

$$u_j = \begin{cases} s_j - t_j x^2 - p_1 & \text{if bought from firm 1,} \\ s_j - t_j(1-x)^2 - p_2 & \text{if bought from firm 2,} \end{cases} \tag{3}$$

where s_j is the value of consumer j on her ideal product, $t_j(\cdot)^2$ represents the transport cost incurred by the consumer in j ($j = H, L$), and p_i is the price set by firm i ($i = 1, 2$). To simplify the analysis, s_j ($j = H, L$) is assumed to be high enough.[19] From (3), a consumer living at $x_j(p_1, p_2)$ is indifferent about firms from which she buys h, where

$$x_j(p_1, p_2) = \frac{p_2 - p_1 + t_j}{2t_j}. \tag{4}$$

3.2 Case I: No firm in the low-end market

In this case, the game becomes a simple duopoly competition a la Hotelling. Each of the consumers in L and H buys one unit of product from firm 1 or 2. From (4),

[19] This assumption guarantees that all consumers buy one unit of a product in equilibrium.

the demand of firm 1, D_1, and that of firm 2, D_2, are given by[20]

$$D_1(p_1,p_2) = \begin{cases} 1 & \text{if } p_2 - p_1 \in [1,+\infty), \\ \lambda x_H(p_1,p_2) + (1-\lambda) & \text{if } p_2 - p_1 \in (t_L, 1), \\ \lambda x_H(p_1,p_2) + (1-\lambda)x_L(p_1,p_2) & \text{if } p_2 - p_1 \in [-t_L, t_L], \\ \lambda x_H(p_1,p_2) & \text{if } p_2 - p_1 \in (-1, -t_L), \\ 0 & \text{if } p_2 - p_1 \in (-\infty, -1], \end{cases} \tag{5}$$
$$D_2(p_1,p_2) = 1 - D_1(p_1,p_2).$$

When $p_2 - p_1 \in [1,+\infty)$ (*resp.* $p_2 - p_1 \in (-\infty,-1]$), x_H and x_L in (4) are larger than 1 (*resp.* smaller than 0), and then D_1 is $\lambda + (1-\lambda) = 1$ (*resp.* 0). When $p_2 - p_1 \in [t_L, 1)$ (*resp.* $p_2 - p_1 \in (-1, -t_L]$), only x_L in (4) is larger than 1 (*resp.* smaller than 0), and then D_1 is $\lambda\, x_H + (1-\lambda)$ (*resp.* $\lambda\, x_H$). When $p_2 - p_1 \in (-t_L, t_L)$, both x_L and x_H are in the range of $(0,1)$.

We first consider whether or not a symmetric Nash equilibrium exists. In this case, the profit functions of the firms are written as:

$$\begin{aligned} \pi_1(p_1,p_2) &= \frac{p_1(t_L + (1-\lambda+\lambda t_L)(p_2-p_1))}{2t_L}, \\ \pi_2(p_1,p_2) &= \frac{p_2(t_L + (1-\lambda+\lambda t_L)(p_1-p_2))}{2t_L}. \end{aligned}$$

The first-order conditions lead to

$$\begin{aligned} p_1' = p_2' &= \frac{t_L}{1-\lambda+\lambda t_L}, \\ \pi_1(p_1',p_2') = \pi_2(p_1',p_2') &= \frac{t_L}{2(1-\lambda+\lambda t_L)}, \end{aligned} \tag{6}$$

where p_i' is the solution of this first-order condition.

Because the profit functions are not concave globally, we have to check whether or not the pair of prices are really the equilibrium outcome. Given $p_1 = t_L/(1-\lambda+\lambda t_L)$ in (6), firm 2 has two options: (i) it sets p_2, which satisfies $p_2 - p_1 \in (t_L, 1)$, and

[20] Rigorously speaking, we must consider cases in which both p_1 and p_2 are so high that some consumers choose not to buy. However, such situations do not occur in an equilibrium because firms never leave any such consumer given a sufficiently high s_j ($j = H, L$). Therefore, we omit the cases.

(ii) it sets p_2, which satisfies $p_2 - p_1 \in (-1, -t_L)$ (see (5)). (i) means that firm 2 gives up supplying to the consumers in L and concentrates on the consumers in H, and (ii) means that firm 2 completely captures the consumers in L (if $p_2 = p_1 - 1$, it completely captures the consumers in L and H).[21] To check whether or not firm 2 actually uses the options, we must solve the following maximization problems:

$$
\begin{aligned}
(i) \qquad & \max_{p_2} \; p_2\lambda(1 - x_H), \quad s.t.\; x_L \geq 1, \\
(ii) \qquad & \max_{p_2} \; p_2(\lambda(1 - x_H) + (1 - \lambda)), \quad s.t.\; x_L \leq 0.
\end{aligned}
$$

In the first problem, the interior solution exists if and only if[22]

$$(1 - \lambda)(1 - 3t_L) - 2\lambda t_L^2 > 0. \tag{7}$$

If the inequality does not hold, the interior solution in (6) is better for firm 2. When the inequality holds, the price and the profit are

$$
\begin{aligned}
p_2'' &= \frac{1 - \lambda + (1 + \lambda)t_L}{2(1 - \lambda + \lambda t_L)}, \qquad (8) \\
\pi_2(p_1', p_2'') &= \frac{\lambda(1 - \lambda + (1 + \lambda)t_L)^2}{8(1 - \lambda + \lambda t_L)}, \qquad (9)
\end{aligned}
$$

where p_i'' is the solution of this first-order condition.

We can easily show that $p_2'' - p_1'$ is smaller than 1.[23] If $\pi_2(p_1', p_2') < \pi_2(p_1', p_2'')$, the pair of prices in (6) is not an equilibrium. This condition is rewritten as

$$(1 - \lambda)(\lambda - 2(2 + \lambda)t_L) > \lambda(3 + \lambda)t_L^2. \tag{10}$$

[21] Firm 2 never sets the price, which satisfies $p_2 \geq p_1 + 1$ (the profit of firm 2 is zero) because this is dominated by $p_2 = p_1$ (the profit is a positive value). The price that satisfies $p_2 < p_1 - 1$ is dominated by $p_2 = p_1 - 1$ because the total quantity supplied by firm 2 is 1 in both cases. Furthermore, in the Appendix, we show that no pure-strategy equilibrium exists, such that $p_i - p_j \in (t_L, 1)$ $(i \neq j)$. Therefore, we can say that no asymmetric pure-strategy equilibrium exists.

[22] This condition is derived from the following procedure. First, assuming $p_1 = p_1'$, solve the first-order condition. Then, substitute the obtained solution p_2'' in (8) and p_1' into x_L. Finally, we check whether $x_L \geq 1$ is satisfied.

[23] The following inequality is always satisfied: $p_2'' - p_1' = (1 - \lambda)(1 - t_L)/(2(1 - \lambda + \lambda t_L)) < 1$.

In the second problem, no interior solution exists (the constraint is always binding). This means that firm 2 never uses this option.

To sum up, if at least one condition ((7) or (10)) does not hold, (p_1', p_2') becomes an equilibrium. Furthermore, because (7) is automatically satisfied if (10), the conclusive condition is

$$(1-\lambda)(1-3t_L) \leq 2\lambda t_L^2. \tag{11}$$

3.3 Case II: Many firms in the low-end market

In this subsection, we assume for simplicity that there are multiple firms that produce l products at each end of the line. Therefore, p^l falls to zero, and no major firm wants to sell its product to the consumers in L.

In this case, the game becomes a simple duopoly game with $D_1 = \lambda x_H(p_1, p_2)$ and $D_2 = \lambda(1 - x_H(p_1, p_2))$. The profit functions of the firms are given by

$$\begin{aligned} \pi_1(p_1, p_2) &= \frac{\lambda p_1(p_2 - p_1 + 1)}{2}, \\ \pi_2(p_1, p_2) &= \frac{\lambda p_2(p_1 - p_2 + 1)}{2}. \end{aligned}$$

Simple calculations show that

$$p_1^* = p_2^* = 1, \quad \pi_1(p_1^*, p_2^*) = \pi_2(p_1^*, p_2^*) = \frac{\lambda}{2}, \tag{12}$$

where p_i^* is firm i's equilibrium price.

3.4 Comparison

Using the results obtained so far, we determine the condition under which case II is more profitable than case I for the two major firms. From equations (6) and (12), the condition is as follows:

$$\frac{\lambda}{2} > \frac{t_L}{2(1-\lambda+\lambda t_L)} \quad \Leftrightarrow \quad t_L < \frac{\lambda}{1+\lambda}. \tag{13}$$

We have to add the condition in (11). Therefore, we obtain the following result.

Proposition 2 *An adequate supply of l is beneficial to the two major firms if and only if the conditions in (11) and (13) are satisfied.*

[Figure 3 here]

The rough intuition of Proposition 2 is similar to that of Proposition 1. If there is no low-end firm and the prices of both major firms are high, each one has an incentive to lower its price because the slight price reduction causes many price-sensitive consumers to switch from the other major firm. Once such a price collapse effect begins to work, the prices of both major firms fall drastically. As a result, the price collapse outweighs the increase in sales volume. If there are low-end firms and p^l is sufficiently low, major firms can avoid a price collapse from the beginning.

Before we proceed to the last section, we should mention the relationship between Chen and Riordan (2007) and our paper. Both papers show that an additional entry might enhance the profits of the incumbent firms.[24]

Their spokes model is a monopolistic competition model with the structure of horizontal differentiation a la Hotelling (1929). The rough structure of their model can be described as follows. Potentially, there are N products in the market. There are $n(\leq N)$ incumbent firms. Firm i, which produces product i ($i = 1, 2, \ldots, n$), faces type i consumers, who are potential buyers of product i. Type i consumers are classified into $N-1$ groups according to another preferred product, $j \neq i$. Type i consumers in group j are uniformly distributed on a Hotelling line. Those consumers never buy the other $N-2$ products. Type i consumers in group j are under one of the following situations: (i) since both products i and j are available, the situation is a standard Hotelling duopoly with firms i and j at the edges of the line; (ii) since

[24] As reported in the following, their model is price competition. Therefore, it would be suitable to compare their model and our price competition model in this section.

only product h ($h \in \{i, j\}$) is available, the situation is a monopoly with firm h at the edge of the line; (iii) since neither product i nor j is available, the situation is an empty market.

[Figure 4 here]

Therefore, there are $N(N-1)/2$ Hotelling lines in the model.[25] The n firms simultaneously determine their prices. Each consumer buys, at most, one good.[26]

Because an additional entry makes the group under (ii) shift to that under (i), the willingness to pay of consumers in the group uniformly decreases (assuming the entrant will choose a reasonable price). Thus, with an increase in the number of entrants, the demand function that an incumbent firm faces becomes as follows: the amount of consumers with relatively high willingness to pay decreases, and that of consumers with relatively low willingness to pay increases. In other words, the demand function becomes a kind of convex one: the demand is inelastic in the (relatively) high-price region, while it is elastic in the (relatively) low-price region. We call this property of elasticity the shift effect.

On the other hand, if all other incumbent firms raise their prices, the demand function that a firm faces shifts upward (partially at the relatively low-price region). This is because the willingness to pay of consumer groups with (i) increases uniformly by the price rise of other preferred products.[27] We call this property the price effect. Because the number of groups with (i) for an incumbent firm is equal to the number of other firms in the market, the price effect increases as the number of firms increases.

[25] There are N types of consumers. Type i consumers are classified into $N-1$ groups. However, type i consumers in group j are equivalent to type j consumers in group i. Therefore, there are $N(N-1)/2$ Hotelling lines.

[26] In the following, we limit our attention to the set of parameters (region III in their paper), from which the result is derived.

[27] It is noteworthy that there is no change for groups with (ii).

One of the possible explanations for their result is that it holds when the price effect dominates the shift effect.[28]

On the contrary, our model has only a single consumer group with (i). Therefore, the logic in Chen and Riordan (2007) does not work. In our model, the additional dimension of consumer heterogeneity (H and L) is the driving force for the result. If there is no firm in the low-end market, firms in the high-end market cannot maintain high prices because each firm has an incentive to obtain a lot of consumers in L (*i.e.,* price-sensitive consumers).[29] Entry in the low-end market benefits the high-end firms by making the low-end market unprofitable for the high-end firms.

Thus, although Chen and Riordan (2007) and our paper argue the same statement that entries might enhance incumbent firms' profits, we investigate quite different situations from those of Chen and Riordan (2007). Furthermore, the driving force of our result is different from that of theirs. Nevertheless, it is possible to say that our results are complementary to those of Chen and Riordan (2007) in the sense that the apparently paradoxical results can be sustained by another logic.[30]

4 Discussion and Concluding Remarks

In this paper, we show that the presence of low-end firms might be beneficial for incumbent high-end firms. The existence of low-end firms induces high-end firms to sell their products only to high-end consumers. The resulting price increase

[28] In order to make the price effect work, firms raise their prices in response to an additional entry. In fact, this occurs because an additional entry reduces the number of distant consumers in the groups with (ii), which reduces each firm's incentive to cut its price in order to capture such distant consumers.

[29] This is essentially the same as the type (i) (prisoners' dilemma) effect in Section 2.

[30] Because the structure of competition is different, it is difficult to compare directly our quantity competition model in the previous section with that of Chen and Riordan (2007). Some readers might think that we can obtain a similar result if we investigate the relationship between the number of high-end firms (without low-end firms) and profitability. Unfortunately, we could not obtain the desired property from our quantity model. See the Appendix for details.

outweighs the resulting sales loss. We show this result in both Cournot and Bertrand models.

In Section 2, we use linear demand functions for simplicity. The linear demand functions allow low-end consumers to make the entire demand function so elastic in the region of low prices that the resulting price collapse outweighs the sales effect. However, we conjecture that we can obtain this property with other functional forms, for example, with certain demand functions that are *concave* in the high-price region and *convex* in the low- price region.[31] When the former property holds, the equilibrium price in the high-end market tends to be higher if the low-end market does not exist. When the latter property holds, the equilibrium price and profit tend to be lower if the low-end market exists. Combining these two properties, we can say that it is profitable for firms to eliminate the low-end market that induces tough competition among the high-end firms. Although we have not explicitly shown those functions, at least, under demand functions obtained by slightly modifying the linear demand function in section 2, the same result would hold.

Our model can be easily extended to a dynamic game of entry deterrence. Suppose that the output level by incumbent low-end firms is insufficient because of high marginal costs and the high-end firms have incentives to sell their products to low-end consumers. Our result implies that the incumbents might "invite" entries if the incumbents cannot establish subsidiaries that produce low-end products.

Although our simple model is useful to provide a clear explanation, at the same time, it is a little specific. In reality, there must be various other situations and effects that produce the same results as ours. Therefore, the construction of other frameworks would be a worthy undertaking for future research.

[31] It is noteworthy that our linear demand function satisfies this property.

Appendix

Proof of the duopoly case in Section 2 In the main part, we have shown that, when no firm produces product l, two types of equilibria exist: $q^x = (1/3, 1/3)$ and $q^y = ((1+b)/3, (1+b)/3)$. We now show the calculus to derive the Lemmas 1 and 2. We must check not only the local optimality (i.e., first-order condition) but also the global optimality. First of all, we define $p^x(q) = 1 - q_1 - q_2$ and $p^y(q) = a(1 + b - (q_1 + q_2))/(a + b)$, which are used later.

Proof of Lemma 1 First, we investigate whether or not a firm deviates from q^y. Without loss of generality, we assume that firm 1 deviates. More concretely, we derive the optimal output level q_1' against q_2^y under $p^x(q)$. If $q_1' \geq 1 - a - q_2^y$, the actual price is given by $p^y(q)$. This implies that firm 1 never deviates. Moreover, if $q_1' < 1 - a - q_2^y$ and $\pi_1(q_1', q_2^y) \leq \pi_1(q^y)$, q^y is a Cournot equilibrium.

A simple calculation shows $q_1' = (2 - b)/6$. Therefore, $q_1' \geq 1 - a - q_2^y$ can be rewritten as $a \geq (2-b)/6$. If $a < (2-b)/6$, $\pi_1(q_1', q_2^y) = (1 - q_1' - q_2^y)q_1' = (2-b)^2/36$. Because $\pi_1(q^y) = a(1+b)^2/9(a+b)$, $\pi_1(q_1', q_2^y) \leq \pi_1(q^y)$ falls into $a \geq (2-b)^2/(3(4+b))$.

When $b \geq 2$, $q_1' \geq 1 - a - q_2^y$ holds for any a. When $b < 2$, if $a \geq (2-b)/6$ or $(2-b)^2/(3(4+b)) < a < (2-b)/6$ holds, q^y is a Cournot equilibrium. When $b < 2$, $(2-b)^2/(3(4+b)) < (2-b)/6$ always holds. Thus, we obtain Lemma 1. Q.E.D.

Proof of Lemma 2 In an analogous way, we can determine the condition under which q^x becomes a Cournot equilibrium. We derive the optimal output level q_1' against q_2^x under $p^y(q)$. If $q_1' \leq 1 - a - q_2^x$, the actual price is given by $p^x(q)$. This implies that firm 1 never deviates. Moreover, if $q_1' > 1 - a - q_2^y$ and $\pi_1(q_1', q_2^x) \leq \pi_1(q^x)$, q^x is a Cournot equilibrium. A simple calculation shows that $q_1' = (2+3b)/6$ and $\pi_1(q_1', q_2^x) = a(2+3b)^2/(36(a+b))$. $q_1' \leq 1 - a - q_2^x$ is rewritten by $a \leq (2-3b)/6$.

$q_1' > 1-a-q_2^x$ and $\pi_1(q_1', q_2^x) \le \pi_1(q^x)$ are rewritten by $(2-3b)/6 < a \le 4/(3(4+3b))$. Because $(2-3b)/6 < 4/(3(4+3b))$ always holds, the condition is written by $a < 4/(3(4+3b))$. Thus, we obtain Lemma 2. Q.E.D.

Duopoly with a single low-end firm We consider the case in which there is a single low-end firm, firm 3 (with two major firms, firm 1 and firm 2). The result is similar to that in Section 2.

Potentially, there are two Nash equilibria. One is the equilibrium with $p^h > p^l$, and the other is the one with $p^h = p^l$. The former one occurs when $q_1 + q_2 < 1 - a$, and the latter occurs otherwise.

In the following, we first derive the condition under which the equilibrium with $p^h > p^l$ exists. Then, we derive the condition under which the existence of a low-end firm benefits the major firms.

If $p^h > p^l$ at the equilibrium, the relevant demand function for the two major firms must be $D^H(p^h) = 1 - p^h$. Therefore, $q_1 = q_2 = 1/3$ becomes the equilibrium after a simple calculation. The equilibrium profit becomes 1/9 for each major firm, and $p^h = 1/3$. Because a low-end firm plays as the monopolist in the low-end market (the demand function is $D^L(p^l) = b(1 - p^l/a)$), $q_3 = b/2$, $p^l = a/2$, and the equilibrium profit becomes $ab/4$. Therefore, if $a < 2/3$, $p^h > p^l$. We now denote this equilibrium ($(q_1, q_2, q_3) = (1/3, 1/3, b/2)$) as q^E.

We now check whether or not q^E is an equilibrium. If q^E is an equilibrium, it must be unprofitable for each major firm to deviate so that $p^h = p^l$. To equalize p^h and p^l, one of the major firms (we call it firm 1) must produce at least[32]

$$q_1^D \equiv \frac{2}{3} - \frac{a}{2}.$$

[32] Given $q_2 = 1/3$, $p^h = 1 - q_1 - q_2 = 2/3 - q_1$. Because $2/3 - q_1 \le p^l = a/2$ is required to make p^h and p^l equalized, firm 1 must produce at least $2/3 - a/2$.

Differentiating $\pi_1(q)$ with respect to q_1 and substituting $q_2 = 1/3$ and $q_3 = b/2$ into it, we have

$$\begin{aligned} & \frac{\partial}{\partial q_1} \left(\frac{a(1+b-\sum_{j=1}^3 q_j)q_1}{a+b} \right) \Bigg|_{q_2=\frac{1}{3}, q_3=\frac{b}{2}} \\ = \; & \frac{a}{a+b} \left(\frac{2}{3} + \frac{b}{2} - 2q_1 \right). \end{aligned}$$

If $a \leq 2/3 - b/2$, this is negative for any $q_1 \geq q_1^D$, and then firm 1 does not have an incentive to deviate. Otherwise, we have the interior solution from it:

$$q_1 = \frac{1}{3} + \frac{b}{4}.$$

In this case, the profit of firm 1 is

$$\frac{a}{a+b} \left(\frac{4+3b}{12} \right)^2.$$

Therefore, firm 1 does not deviate if

$$\frac{a}{a+b} \left(\frac{4+3b}{12} \right)^2 < 1/9.$$

After some calculations, this can be rewritten as

$$a < \frac{16}{3(8+3b)}.$$

Therefore, if $a \leq \max\{2/3 - b/2, 16/(3(8+3b))\}$ holds, no major firm wants to deviate. Because $2/3 - b/2 < 16/(3(8+3b)$ is satisfied, we obtain the following lemma.

Lemma 4 *If $a < 16/(3(8+3b))$, there exists an equilibrium such that the two major firms obtain $1/9$.*

Now, we derive the condition under which the existence of a low-end firm benefits the major firms. We need one more condition for this result: each major firm actually benefits from the entry of firm 3. This condition is

$$\frac{1}{9} \geq \frac{a(1+b)^2}{9(a+b)} \quad \Leftrightarrow \quad \frac{1}{2+b} \geq a.$$

Furthermore, a simple calculation shows that $a < 16/(3(8+3b))$ is automatically satisfied if $a(2+b) \leq 1$ holds (*i.e.*, $16/(3(8+3b)) > 1/(2+b)$). Therefore, we obtain the desired result: the existence of a low-end firm benefits the major firms if $a \leq 1/(2+b)$. (It is noteworthy that this condition is the same as the one in Proposition 1.) Q.E.D.

Proof of pricing equilibrium in Bertrand competition We now show that there exists no pure strategy equilibrium such that $p_2 - p_1$, $(-\infty, -t_L] \cup [t_L, +\infty)$. To discuss the matter, we rewrite the quantities supplied by the firms:

$$D_1(p_1,p_2) = \begin{cases} 1 & \text{if } p_2 - p_1 \in [1, +\infty), \\ \lambda x_H(p_1,p_2) + (1-\lambda) & \text{if } p_2 - p_1 \in (t_L, 1), \\ \lambda x_H(p_1,p_2) + (1-\lambda) x_L(p_1,p_2) & \text{if } p_2 - p_1 \in [-t_L, t_L], \\ \lambda x_H(p_1,p_2) & \text{if } p_2 - p_1 \in (-1, -t_L), \\ 0 & \text{if } p_2 - p_1 \in (-\infty, -1], \end{cases} \tag{5'}$$

$$D_2(p_1,p_2) = 1 - D_1(p_1,p_2).$$

In the first and fifth cases ($p_2 - p_1 \in [1, +\infty)$ and $p_2 - p_1 \in (-\infty, -1]$), it is clear that no such pure strategy equilibrium exists. When those cases appear, one of the firms does not supply because it sets its price too high. If the firm setting a higher price equalizes its price to the rival's, it earns a positive profit.

We now investigate the second and fourth cases ($p_2 - p_1 \in (t_L, 1)$ and $p_2 - p_1 \in (-1, -t_L)$). By symmetry, we only discuss the fourth case ($p_2 - p_1 \in (-1, -t_L)$). The profit functions of the firms are described as follows.

$$
\begin{aligned}
\pi_1(p_1, p_2) &= \frac{\lambda p_1(p_2 - p_1 + 1)}{2} \\
\pi_2(p_1, p_2) &= p_2\left(\frac{\lambda(p_1 - p_2 + 1)}{2} + (1+\lambda)\right) \quad ((p_1, p_2) \in (-1, -t_L])
\end{aligned}
$$

Now, we suppose that there exists a pure strategy equilibrium (p_1^*, p_2^*) and derive the contradiction that $p_2^* - p_1^* \notin (-1, -t_L)$.

Because no corner solution is allowed in this region $(p_2 - p_1 \in (-1, -t_L))$, (p_1^*, p_2^*) must satisfy the following first- order conditions if it is an equilibrium.

$$
\begin{aligned}
p_2^* - 2p_1^* + 1 &= 0 \\
\frac{\lambda(p_1^* - 2p_2^* + 1)}{2} + (1 - \lambda) &= 0
\end{aligned}
$$

Solving these equalities yields $(p_1^*, p_2^*) = (\frac{2+\lambda}{3\lambda}, \frac{4-\lambda}{3\lambda})$. However, $p_2^* - p_1^* = \frac{2(1-\lambda)}{3\lambda} > 0$, which means that $p_2^* - p_1^* \notin (-1, -t_L)$. Therefore, no pure strategy equilibrium exists in the second and fourth cases. Q.E.D.

Profits of n high-end firms with no low-end firm We consider a case in which no firm produces l and the $n(\geq 2)$ major firms can potentially sell to both groups of consumers. The purpose of this analysis is to show that each firm's profit is decreasing in the number of firms.

Similarly to (1), p^h and $\pi_i(q)$ are determined as follows.

$$
p^h(q) = \begin{cases} 1 - \sum_{i=1}^{n} q_i & \text{if } \sum_{i=1}^{n} q_i \leq 1 - a, \\ \dfrac{a(1 + b - \sum_{i=1}^{n} q_i)}{a+b} & \text{otherwise.} \end{cases}
$$

$$
\pi_i(q) = \begin{cases} \left(1 - \sum_{i=1}^{n} q_i\right) q_i & \text{if } \sum_{i=1}^{n} q_i \leq 1 - a, \\ \dfrac{a(1 + b - \sum_{i=1}^{n} q_i)q_i}{a+b} & \text{otherwise.} \end{cases} \tag{15}
$$

Depending on (a,b), we have two local optimal solutions: $q_i^A(n) = 1/(n+1)$ for all $i = 1, 2, \cdots, n$ (if $a \leq 1/(n+1)$), and $q_i^B(n) = (1+b)/(n+1)$ for all $i = 1, 2, \cdots, n$ (if $a \geq (1-nb)/(n+1)$). $q_i^A(n)$ and $q_i^B(n)$ are the generalizations of q^x and q^y in Lemmas 1 and 2, respectively. Define $q^j(n) = (q_1^j(n), q_2^j(n), \cdots, q_n^j(n))$ and $Q^j(n) = \sum_{i=1}^n q_i^j(n)$ for $j = A, B$.

If we try to derive the conditions on (a,b) under which $q^A(n)$ ($q^B(n)$) becomes an equilibrium, we have to check whether the local optimal solution is also globally optimal. After some calculus, we have the following lemmas. The proof of the lemmas is given by the supplementary material. The computation is tedious but similar to that in the duopoly case.

Lemma 5 *$q^B(n)$ becomes a Cournot equilibrium if $a \geq (2-(n-1)b)^2/(n+1)(4-(n-3)b)$ or $b \geq 2/(n-1)$. Each firm obtains $\pi^B(n) \equiv a(1+b)^2/(a+b)(n+1)^2$.*

Lemma 6 *$q^A(n)$ becomes a Cournot equilibrium if $a \leq 4/(n+1)(4+(n+1)b)$ is satisfied. Each firm obtains $\pi^A(n) \equiv 1/(n+1)^2$.*

Basically, as n increases, the equilibrium pattern (for a given (a,b)) shifts (i) $\rightarrow$ (ii) $\rightarrow$ (iii), where (i) q^A is the only equilibrium, (ii) both q^A and q^B are the equilibria, and (iii) q^B is the only equilibrium. (It is noteworthy that (i) and (ii) do not appear regardless of n when a and/or b is large because the low-end market is highly profitable; see Lemmas 5 and 6.) To see this, suppose temporarily that $p^h(q) = 1 - \sum_{i=1}^n q_i$ for all q. Under this demand function, the total equilibrium output level $Q^A(n)$ is increasing in n and $\lim_{n\to\infty} Q^A(n) = 1$. This implies that $q^A(n)$ cannot be an equilibrium for all $n > \hat{n}$, where $\hat{n}$ is the minimal n such that $Q^A(n) > 1-a$. Analogously, q^B cannot be an equilibrium for n such that $Q^B(n) < 1-a$.

Based on this equilibrium pattern, we show that each firm's profit is decreasing in a number of firms. First, both $\pi^A(n)$ and $\pi^B(n)$ in the above lemmas are decreasing functions in n. Second, a firm's profit decreases at the switch from $q^A(n)$ to $q^B(n)$ in the region of (ii). The sign of $\pi^A(n) - \pi^B(n)$ is independent of n because

$$\pi^A(n) - \pi^B(n) = \frac{b(1 - a(2+b))}{(a+b)(n+1)^2}.$$

Therefore, it is sufficient to show that $1 - a(2+b) \geq 0$ as long as $q^A(n)$ is an equilibrium. In other words, when $1-a(2+b) < 0$, only (iii) appears. This is derived from the condition of Lemma 6. Given that the RHS of the condition decreases in n and substituting $n = 1$ into the condition yields $a \leq 1/(2+b)$, $1 - a(2+b)$ must be non-negative if $q^A(n)$ is an equilibrium.

Therefore, we obtain the desired result. (However, the profit might increase if we artificially choose particular equilibria in the region of (ii). For example, select $q^B(n)$ as the equilibrium for n and $q^A(n+1)$ as the equilibrium for $n+1$. Under certain (a, b), it might be possible to find n such that $\pi^B(n) < \pi^A(n+1)$.)

References

[1] Baye, Michael R., Morgan, John, and Scholten, Patrick, 2004, Price dispersion in the small and the large: Evidence from an Internet price comparison site, Journal of Industrial Economics 52, 463–496.

[2] Cabral, Luís M. B., 2000, Introduction to industrial organization, Cambridge, MA: MIT Press.

[3] Chen, Yongmin, and Riordan, Michael H., 2007, Price and variety in the spokes model, Economic Journal 117, 897–921.

[4] Coughlan, Anne T., and Soberman, David A., 2005, Strategic segmentation using outlet malls, International Journal of Research in Marketing 22, 61–86.

[5] Davis, Steven J., Murphy, Kevin M., and Topel, Robert H., 2004, Entry, pricing, and product design in an initially monopolized market, Journal of Political Economy Part 2 Supplement 112, S188–225.

[6] Frank, Richard G., and Salkever, David S., 1997, Generic entry and the pricing of pharmaceuticals, Journal of Economics and Management Strategy 6, 75–90.

[7] Hauser, John R., and Shugan, Steven M., 1983, Defensive marketing strategies, Marketing Science 2, 319–360.

[8] Hotelling, Harold, 1929, Stability in competition, Economic Journal 39, 41–57.

[9] Inderst, Roman, 2002, Why competition may drive up prices, Journal of Economic Behavior and Organization 47, 451–462.

[10] Keller, Kevin Lane and Lehmann, Donald R., 2006, Brands and branding: Research findings and future priorities, Marketing Science 25, 740–759.

[11] Leclerc, France, Christopher K. Hsee and Nunes, Joseph C., 2005, Narrow focusing: Why the relative position of a good in its category matters more than it should, Marketing Science 24, 194–205.

[12] Narasimhan, Chakravarthi, 1988, Competitive promotional strategies, Journal of Business 61, 427–449.

[13] Pauwels, Koen, and Srinivasan, Shuba, 2004, Who benefits from store brand entry? Marketing Science 23, 364–390.

[14] Randall, Taylor, Ulrich, Karl, and Reibstein, David, 1998, Brand equity and vertical product line extent, Marketing Science 17, 356–379.

[15] Rosenthal, Robert W., 1980, A model in which an increase in the number of sellers leads to a higher price, Econometrica 48, 1575–1579.

[16] Satterthwaite, Mark A., 1979, Consumer information, equilibrium industry price, and the number of sellers, Bell Journal of Economics 10, 483–502.

[17] Sayman, Serdar, Hoch, Stephen J., and Raju, Jagmohan S., 2002, Positioning of store brands, Marketing Science 21, 378–397.

[18] Soberman, David A., and Parker, Philip M., 2004, Private labels: Psychological versioning of typical consumer products, International Journal of Industrial Organization 22, 849–861.

[19] Steenkamp, Jan-Benedict E.M., Nijs, Vincent R., Hanssens, Dominique M., and Dekimpe, Marnik G., 2005, Competitive reactions to advertising and promotion attacks, Marketing Science 24, 35–54.

[20] Stiglitz, Joseph E., 1987, Competition and the number of firms in a market: Are duopolies more competitive than atomistic markets? Journal of Political Economy 95, 1041–1061.

[21] Schulz, Norbert, and Stahl, Konrad, 1996, Do consumers search for the highest price? Oligopoly equilibrium and monopoly optimum in differentiated-products markets, RAND Journal of Economics 27, 542–562.

[22] Ward, Michael B., Shimshack, Jay P., Perloff, Jeffrey M., and Harris, J. Michael, 2002, Effects of the private-label invasion in food industries, American Journal of Agricultural Economics 84, 961–973.

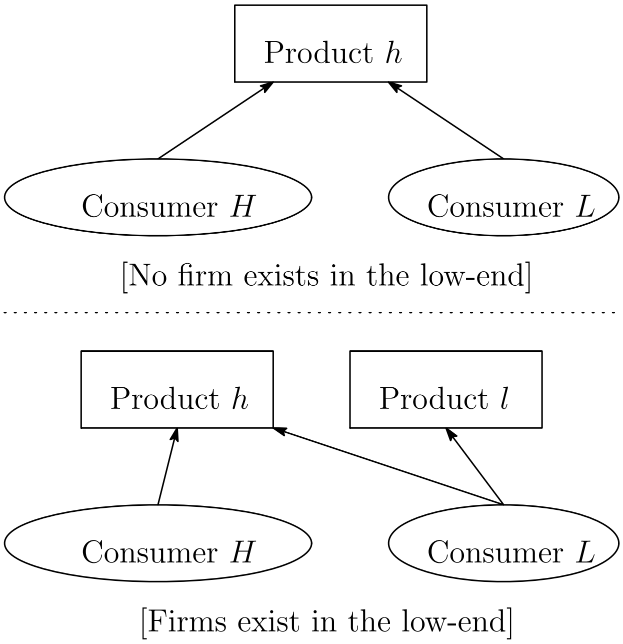

Figure 1: The market structure

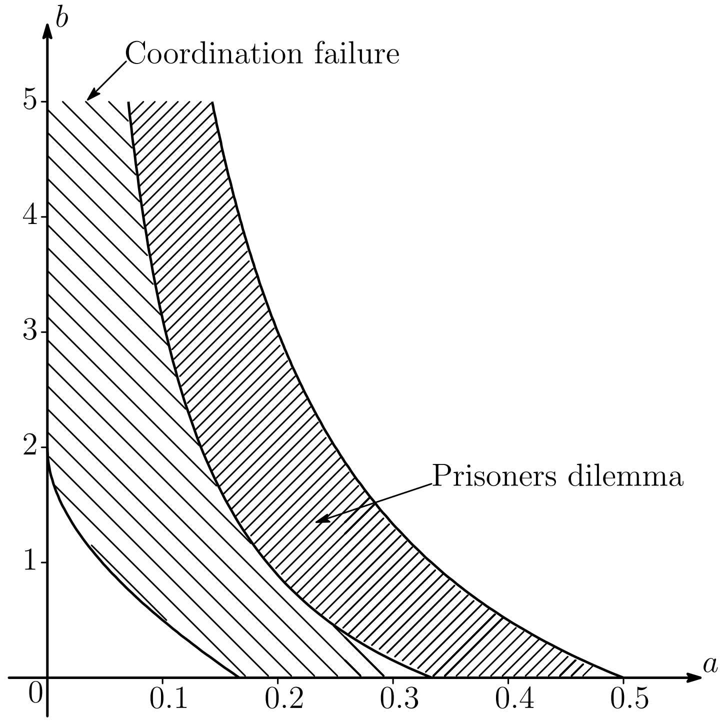

Figure 2: The parameter range within which the handover is beneficial.

(Horizontal: a, Vertical: b)

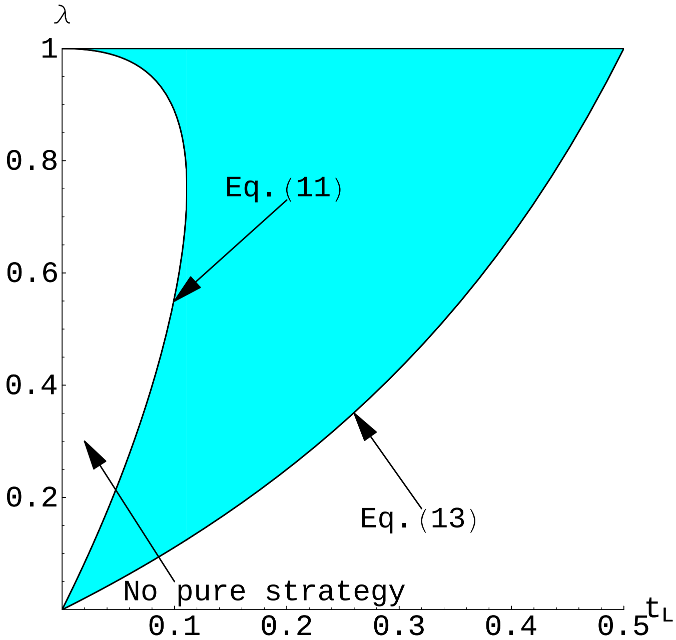

Figure 3: The parameter range within which the handover is beneficial.
(Horizontal: t_L, Vertical: λ)

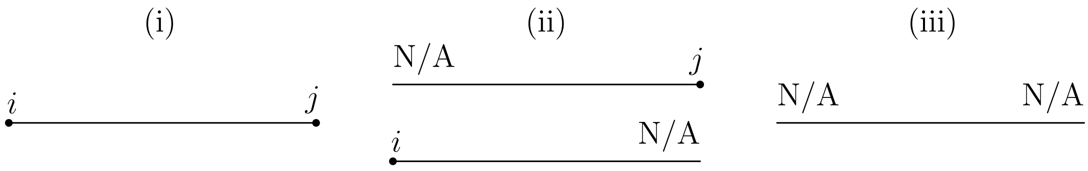

Figure 4: The market structure for type i consumers in group j.