Proof of Theorem 10.1 in Tadelis (2012)

To make the proof, we follow the proof in Fudenberg and Tirole (1991, pp. 154-5). To clarify the discussion, we write several sentences which are not needed to formally prove it.

Assume that there is a pure action profile $a = (a_1, a_2, \ldots, a_n)$ such that $v(a) = (v_1, v_2, \ldots, v_n)$.

Consider the following strategy of player i: "Play a_i in period 0, and continue to play a_i as long as either (i) the realized action in the previous period was a or (ii) the realized action in the previous period differed from a in two or more components. If in some previous period, player i was the only one not to follow profile a, then each player j ($j \neq i$) plays m_j^i for the rest of the game, where m_j^i is

$$\underline{v}_i = \min_{\alpha_{-i}} \left[\max_{\alpha_i} v_i(\alpha_i, \alpha_{-i}) \right],$$

so called the minmax profile against player i. That is, all the players except player i employ the actions which minimize player i's payoff, anticipating that that player i maximizes its own payoff.

In the period in which player i deviates, he receives at most $\max_a v_i(a)$, and then he receives at most $\underline{v}_i$ in periods after his first deviation. If he deviates in period t, he obtains at most

$$\Pi_D \equiv \frac{1-\delta^t}{1-\delta} v_i + \delta^t \max_a v_i(a) + \frac{\delta^{t+1}}{1-\delta} \underline{v}_i. \tag{1}$$

The first term is the present value of the payoffs from period 1 to $t-1$; the second term is the present value of the payoff in period t; the third term is the present value of the payoffs from period $t+1$.

Π_D in (1) is less than $v_i/(1-\delta)$, which is achieved when no one deviates, if δ is larger than the critical value $\underline{\delta}_i$ such that

$$(1-\underline{\delta}_i) \max_a v_i(a) + \underline{\delta}_i \underline{v}_i = v_i.$$

This is derived by the condition that $\Pi_D = v_i(1-\delta)$ (we replace δ with $\underline{\delta}_i$). Because $v_i > \underline{v}_i$, $\underline{\delta}_i$ is less than 1. By taking $\underline{\delta} = \max_i \underline{\delta}_i$, we complete the argument. Q.E.D.

Reference

Fudenberg, Drew, and Tirole, Jean. 1991. *Game Theory.* MIT Press. Cambridge, Massachusetts.