# Chapter 3: Preliminaries

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   What is "simultaneous"?

   We do NOT require that players all choosing their actions at the *exact same* moment.

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1. Each player *simultaneously* and *independently* chooses an action.
   Players must take their actions *without observing* what actions their counterparts take and *without interacting* with other players to coordinate their actions.

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2. Conditional on the players' choices of actions, payoffs are distributed to each player.
   That is, once the players have all made their choices, these choices will result in a particular outcome, or probabilistic distribution over outcomes

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1. everyone knows $E$,
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1. all the possible actions of all the players,
2. all the possible outcomes,
3. how each combination of actions of all players affects which outcome will materialize,
4. the preferences of each and every players over outcomes.

# Normal-Form Games

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**Normal-form game** A normal-form game consists of three features:

1. A set of **players**.
2. A set of **actions** for each player.
3. A **set of payoff functions** for each player that give a payoff value to each combination of the players' chosen actions.

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**Strategy** A plan of action intended to accomplish a special goal.

(ex.) If he asks me question $q_1$ then I will respond with answer $a_1$; if he asks me question $q_2$ then I will respond with answer $a_2$; ...
(note that this is not a static situation).

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**Strategy** A plan of action intended to accomplish a special goal.

**Definition 3.2** A *pure* strategy for player $i$ is a *deterministic* plan of action. The set of all pure strategies for player $i$ is denoted $S_i$. A profile of pure strategies $s = (s_1, s_2, \ldots, s_n)$, $s_i \in S_i$ for all $i = 1, 2, \ldots, n$, describes a particular combination of pure strategies chosen by all $n$ players in the game.

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This is because the players choose actions once and for all and simultaneously.

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Note: Applying the concept of a strategy for a plan of action to a *static game* of complete information is overkill (excessive).

In a static situation, a player's (pure) strategy is a plan that he/she chooses an action (with probability one). Given the strategy, he/she chooses the action.

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2. A collection of sets of pure strategies, $\{S_1, S_2, \ldots, S_n\}$.
3. A set of payoff functions, $\{v_1, v_2, \ldots, v_n\}$, each assigning a payoff value to each combination of chosen strategies.

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1. A finite set of players, $N = \{1, 2, \ldots, n\}$.
2. A collection of sets of pure strategies, $\{S_1, S_2, \ldots, S_n\}$.
3. A set of functions $v_i$: $S_1 \times S_2 \times \cdots \times S_n \to \Re$ for each $i \in N$.

# Examples

**Normal-form game** A triple of sets: $\langle N, \{S_i\}_{i=1}^n, \{v_i(\cdot)\}_{i=1}^n \rangle$, where $N$ is the set of players, $\{S_i\}_{i=1}^n$ is the set of all players' strategy sets, and $\{v_i(\cdot)\}_{i=1}^n$ is the set of all players' payoff functions over the strategy profile of all the players.

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**Example: The prisoner's dilemma** There are two men who are suspects in a serious crime (armed robbery). The police have evidence of only petty theft, and to nail the suspects for the armed robbery that need testimony from at least one of the suspects.

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**Example: The prisoner's dilemma** There are two men who are suspects in a serious crime (armed robbery). The police separates the two suspects at the police station and questions each in a different room.

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**Example: The prisoner's dilemma** There are two men who are suspects in a serious crime (armed robbery). Each suspect is offered a deal that reduces the sentence he will get if he confesses, or "finks" ($F$). The alternative is for the suspect to say nothing to the investigators, or remain "mum" ($M$).

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**Example: The prisoner's dilemma** There are two men who are suspects in a serious crime (armed robbery).
Each suspect: "finks" ($F$) or "mum" ($M$).
The payoff of each suspect:
If both choose $M$, both get 2 years in prison.
If both choose $F$, both get 4 years in prison.
If player 1 (2) chooses $M$ ($F$), player 1 gets 5 years (1 year) in prison.

# Examples

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**Example: The prisoner's dilemma**

Each suspect: "finks" $(F)$ or "mum" $(M)$.
If both choose $M$ $(F)$, both get $-2$ $(-4)$.
If player 1 (2) chooses $M$ $(F)$, player 1 gets $-5$ $(-1)$.

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**Formal description** This game is written as
$N = \{1, 2\}$. $S_i = \{M, F\}$ for $i \in \{1, 2\}$.

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**Formal description** This game is written as

$N = \{1, 2\}$. $S_i = \{M, F\}$ for $i \in \{1, 2\}$.

Let $v_i(s_1, s_2)$ be the payoff to player $i$ if player 1 chooses $s_1$ and player 2 chooses $s_2$.

$$v_1(M, M) = v_2(M, M) = -2,$$
$$v_1(F, F) = v_2(F, F) = -4,$$
$$v_1(M, F) = v_2(F, M) = -5,$$
$$v_1(F, M) = v_2(M, F) = -1.$$

# Examples

**Cournot duopoly** Two identical firms, players 1 and 2, produce homogenous goods. The cost function of player $i$ is $c_i(q_i) = q_i^2$ ($i \in \{1, 2\}$), where $q_i$ is its quantity. Each firm simultaneously sets its quantity. The price in the market, $p$, is determined by

$$p = \begin{cases} a - (q_1 + q_2) & \text{if } q_1 + q_2 < a, \\ 0 & \text{if } q_1 + q_2 \geq a. \end{cases}$$

The objective of each player is to maximize its own profit.

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**The normal form** The scenario is summarized as follows:

$N = \{1, 2\}$,

$S_i = [0, \infty)$ for $i \in \{1, 2\}$ and firm $i$ chooses $s_i \in S_i$,

For $i, j \in \{1, 2\}$, $i \neq j$,

$$v_i(s_i, s_j) = \begin{cases} (a - (s_i + s_j))s_i - s_i^2 & \text{if } s_i + s_j < a, \\ -s_i^2 & \text{if } s_i + s_j \geq a. \end{cases}$$

## 3.2 Matrix Representation: Two-Player Finite Game

In many cases, players may be constrained to choose one of a finite number of actions (e.g., Prisoner's dilemma).

# Matrix Representation

**Definition 3.4** A finite game is a game with a finite number of players, in which the number of strategies in $S_i$ is finite for all players $i \in N$.

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Any two-player finite game can be represented by a matrix that will capture all the relevant information of the normal-form game.

- Rows: Player 1's strategies.
- Columns: Player 2's strategies.
- Matrix entries: Each entry in this matrix contains a two-element vector $(v_1, v_2)$, where $v_i$ is player $i$'s payoff.

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- Rows: Player 1's strategies.
- Columns: Player 2's strategies.
- Matrix entries: $(v_1, v_2)$.

**Prisoner's dilemma** Each player has two actions, $F$ and $M$.

Player 2

| Player 1 | | $F$ | $M$ |
|---|---|---|---|
| | $F$ | $(-4, -4)$ | $(-1, -5)$ |
| | $M$ | $(-5, -1)$ | $(-2, -2)$ |

## 3.3 Solution Concepts

Example: Prisoner's dilemma

| | | Player 2 | |
|---|---|---|---|
| | | $F$ | $M$ |
| Player 1 | $F$ | $(-4,-4)$ | $(-1,-5)$ |
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# Solution Concepts

Example: Prisoner's dilemma

| | | Player 2: $F$ | Player 2: $M$ |
|---|---|---|---|
| Player 1 | $F$ | $(-4, -4)$ | $(-1, -5)$ |
| | $M$ | $(-5, -1)$ | $(-2, -2)$ |

For player $i$, choosing $F$ is better irrespective of the opponent's action.

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| Player 1 | $F$ | $(-4, -4)$ | $(-1, -5)$ |
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For player $i$, choosing $F$ is better irrespective of the opponent's action.
Note that, the payoffs are based on the assumption that the longer the period in prison, the worse the payoff of the player.

# Solution Concepts

Example: Prisoner's dilemma

| | | Player 2 | |
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| | | $F$ | $M$ |
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For instance, if each player takes into account not only *its own period* but also *his/her opponent's period*, the payoffs change, which may change the prediction to what will happen (see p.53).

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For instance, if each player takes into account not only *its own period* but also *his/her opponent's period*, the payoffs change, which may change the prediction to what will happen (see p.53).

This implies that we have to get the game parameters right.

# Solution Concepts

Example: The Battle of the Sexes

| | | Chris | |
|---|---|---|---|
| | | $O$ | $F$ |
| Alex | $O$ | $(2, 1)$ | $(0, 0)$ |
| | $F$ | $(0, 0)$ | $(1, 2)$ |

# Solution Concepts

Example: The Battle of the Sexes

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| Alex | $O$ | $(2,1)$ | $(0,0)$ |
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For each player, choosing $F$ (choosing $O$) is NOT always better.

# Solution Concepts

Example: The Battle of the Sexes

| | | Chris | |
|---|---|---|---|
| | | $O$ | $F$ |
| Alex | $O$ | $(2, 1)$ | $(0, 0)$ |
| | $F$ | $(0, 0)$ | $(1, 2)$ |

We need a **solution concept** that will result in predictions or prescriptions.

# Solution Concepts

Example: The Battle of the Sexes

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**Solution concept:** A method of analyzing games with the objective of restricting the set of *all possible outcomes* to those that are *more reasonable than others*.

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An example: Each player chooses the action that is *always best*, regardless of what his/her opponents will choose.

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**Solution concept:** A method of analyzing games with the objective of restricting the set of *all possible outcomes* to those that are *more reasonable than others*.

An example: Each player chooses the action that is *always best*, regardless of what his/her opponents will choose.

This concept is not useful in the above game.

# Solution Concepts

Example: The Battle of the Sexes

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| | | $O$ | $F$ |
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**Equilibrium:** Any one of the strategy profiles that emerges as one of the solution concept's predictions.

## 3.3.1 Assumptions and Setup

Throughout the lecture, we set the following assumptions:

1. Players are "rational": A *rational* player is one who chooses his strategy/action, $s_i \in S_i$, to maximize his/her payoff consistent with his/her beliefs about what is going on in the game.

# Solution Concepts

Throughout the lecture, we set the following assumptions:

1. Players are “rational”:
2. Players are “intelligent”: An *intelligent* player knows everything about the game: the actions, the outcomes, and the preferences of all the players.

# Solution Concepts

Throughout the lecture, we set the following assumptions:

1. Players are "rational":
2. Players are "intelligent":
3. Common knowledge: The fact that players are rational and intelligent is common knowledge among the players of the game.

# Solution Concepts

Throughout the lecture, we set the following assumptions:

1. Players are "rational":
2. Players are "intelligent":
3. Common knowledge:
4. Self-enforcing: Any prediction (or equilibrium) of a solution concept must be self-enforcing.

# Solution Concepts

Throughout the lecture, we set the following assumptions:

1. Players are "rational":
2. Players are "intelligent":
3. Common knowledge:
4. Self-enforcing:
   The fourth requirement is at the heart of **noncooperative game theory**.

# Solution Concepts

Throughout the lecture, we set the following assumptions:

1. Players are “rational”:
2. Players are “intelligent”:
3. Common knowledge:
4. Self-enforcing:
   The fourth requirement is at the heart of **noncooperative game theory**.

   **Noncooperative behavior:** Each player is in control of his/her own actions, and he/she will stick to an action only if he/she finds it to be in his/her best interest.

## 3.3.2 Evaluating Solution Concepts

We must evaluate a theory by how it does as a methodological tool.

# Solution Concepts

The following three criteria is useful to evaluate a variety of solution concepts.

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   Bad example 1: Each player chooses the action that is *always best*, regardless of what his/her opponents will choose.

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The following three criteria is useful to evaluate a variety of solution concepts.

1. Existence: How often does it apply?
   Bad example 2: Players always choose the action that they think their opponent will choose.

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The following three criteria is useful to evaluate a variety of solution concepts.

1. Existence: How often does it apply?
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   A useful concept restricts the set of possible outcomes to *a smaller set* of reasonable outcomes.

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1. Existence: How often does it apply?
2. Uniqueness: How much does it restrict behavior?
   A useful concept restricts the set of possible outcomes to *a smaller set* of reasonable outcomes.
   Bad example: A concept says “everything can happen.”

# Solution Concepts

The following three criteria is useful to evaluate a variety of solution concepts.

1. Existence: How often does it apply?
2. Uniqueness: How much does it restrict behavior?
3. Invariance: How sensitive is it to small changes?

# Solution Concepts

The following three criteria is useful to evaluate a variety of solution concepts.

1. Existence: How often does it apply?
2. Uniqueness: How much does it restrict behavior?
3. Invariance: How sensitive is it to small changes?
   An increase in the number of players is NOT a small change.

# Solution Concepts

The following three criteria is useful to evaluate a variety of solution concepts.

1. Existence: How often does it apply?
2. Uniqueness: How much does it restrict behavior?
3. Invariance: How sensitive is it to small changes?
   An addition/An elimination of strategies is NOT a small change.

# Solution Concepts

The following three criteria is useful to evaluate a variety of solution concepts.

1. Existence: How often does it apply?
2. Uniqueness: How much does it restrict behavior?
3. Invariance: How sensitive is it to small changes?
   A small change in the payoffs of the players.

## 3.3.3 Evaluating Outcomes

The following criterion is often used to evaluate outcomes in games.

**Definition 3.5 (Pareto criterion)** A strategy profile $s \in S$ **Pareto dominates** strategy profile $s' \in S$ if $v_i(s) \geq v_i(s')\ \forall i \in N$ and $v_i(s) > v_i(s')$ for at least one $i \in N$ (in which case, $s'$ is **Pareto dominated** by $s$). A strategy profile is **Pareto optimal** if it is not Pareto dominated by any other strategy profile.

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**Ex: Prisoner's dilemma** The strategy profile $(F, F)$ is Pareto dominated by the strategy profile $(M, M)$.

# Exercises

## 3.5 Exercises

**3.6 Price competition** The market demand is

$$q(p) = 100 - p.$$

Two firms, firms 1 and 2, exist. Each firm $i$ simultaneously chooses its price, $p_i$.

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Write down the normal form of this game.

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$N = \{1, 2\}$; $S_i = [0, \infty)$ and firm $i$ chooses $s_i = [0, \infty)$;
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$v_i(s_i, s_j)$ is given as

$$v_i(s_i, s_j) = \begin{cases} s_i(100 - s_i) & \text{if } s_i < s_j, \\ s_i(100 - s_i)/2 & \text{if } s_i = s_j, \\ 0 & \text{if } s_i > s_j. \end{cases}$$