Chapter 13: Auctions and competitive bidding

Outline

- Independent private values
  Second-price sealed-bid auctions, First-price sealed-bid auctions
- Common values
  The value of a player is correlated to that of another in a second-price auction.
  A answer key for Exercise 13.8

13.1.1 Second-price sealed-bid auctions

Basic setting The set of bidders: $N = \{1, 2, \cdots, n\}$.

Player i's type, θ_i, is drawn from the interval $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$, cdf $F_i(\cdot)$, where $F_i(\theta') = \Pr\{\theta_i \leq \theta'\}$ and $\underline{\theta}_i \geq 0$.

The draws of θ_i are independent and not correlated.

Player i's payoff: $v_i = \begin{cases} \theta_i - p & \text{getting the good,} \\ 0 & \text{not,} \end{cases}$

where p is the price.

Every player knows the distribution functions $F_j(\cdot)$ $(j \neq i)$, which are used to form beliefs about the types θ_{-i} of the other players.

Independent Private Values

Second-price auction Player i sets his bid, b_i. The highest bidder wins, and he pays a price equal to *the second highest bid*.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ \frac{\theta_i - b_i}{\#\text{highest bidders}} & \text{if } b_i \geq b_j \text{ for all } j \neq i \text{ and} \\ & \quad b_i = b_j \text{ for some } j \neq i, \\ 0 & \text{if } b_i < b_j \text{ for some } j \neq i. \end{cases}$$

(Player i's strategy) $s_i : [\underline{\theta}_i, \bar{\theta}_i] \rightarrow \Re_+$.

Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Player i's expected payoff

$$E_{\theta_{-i}}[v_i(b_i; s_{-i}(\theta_{-i}); \theta_i) | \theta_i]$$
$$= \Pr\{i \text{ wins and pays } p\} \times (\theta_i - p) + \Pr\{i \text{ loses}\} \times 0.$$

b_i does not influence p but does the winning prob..

Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b^*_j & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b^*_j \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Player i's expected payoff

$$E_{\theta_{-i}}[v_i(b_i; s_{-i}(\theta_{-i}); \theta_i)|\theta_i]$$
$$= \Pr\{i \text{ wins and pays } p\} \times (\theta_i - p) + \Pr\{i \text{ loses}\} \times 0.$$

b_i does not influence p but does the winning prob..

Proposition 13.1 Each player has a weakly dominant strategy, which is to bid his true valuation. That is, $s_i(\theta_i) = \theta_i$ for all $i \in N$ is a Bayesian Nash equilibrium.

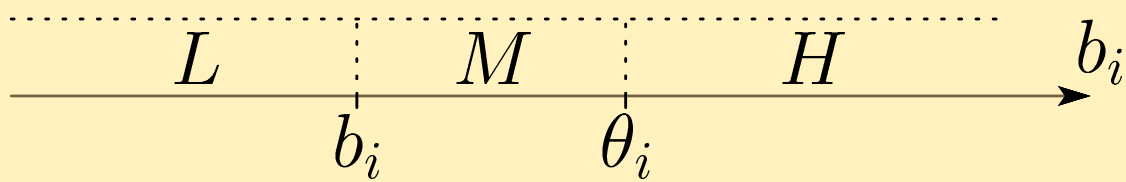
Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Proposition 13.1 $s_i(\theta_i) = \theta_i$ for all $i \in N$.

$(b_i < \theta)$



- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ on L:
  He wins and pays b_j^*. Bidding θ_i leads to the same result.

Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad\text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

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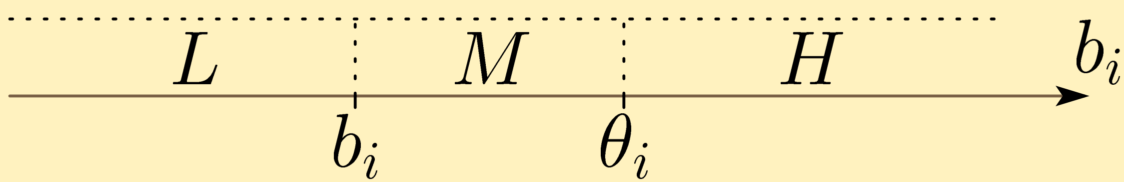

- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ on L:
- b_i is not the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on M: He loses. But, bidding θ_i leads to the win, increasing his expected payoff.

Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Proposition 13.1 $s_i(\theta_i) = \theta_i$ for all $i \in N$.

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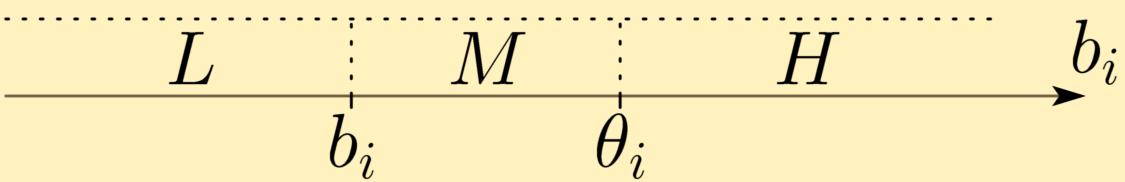


- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ on L:
- b_i is not the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on M:
- b_i is not the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on H:
  He loses. Bidding θ_i leads to the same result.

Bidding θ_i weakly dominates any lower bid.

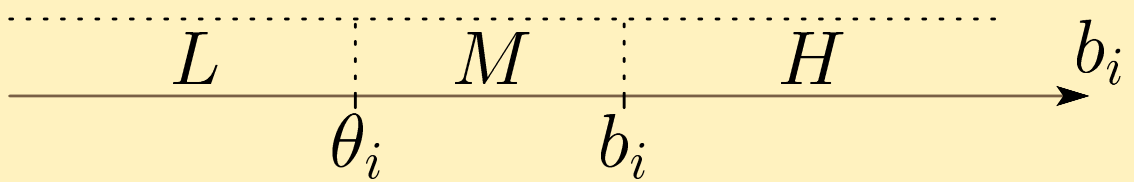
Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Proposition 13.1 $s_i(\theta_i) = \theta_i$ for all $i \in N$.

$(b_i > \theta)$



- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ on L:
  He wins and pays b_j^*. Bidding θ_i leads to the same result.
- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on M:
- b_i is not the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on H:

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Proposition 13.1 $s_i(\theta_i) = \theta_i$ for all $i \in N$.

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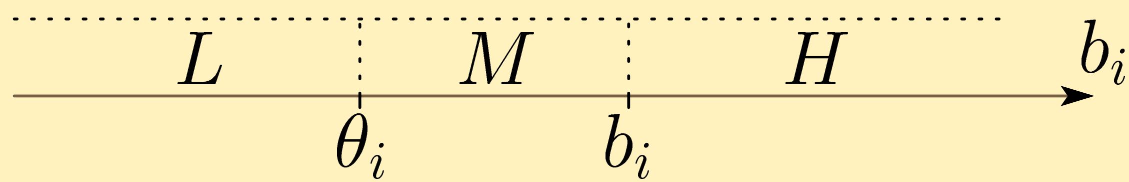


- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ on L:
- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on M:
  He wins and pays $b_j^* (> \theta_i)$. But, bidding θ_i leads to the loss, diminishing his expected payoff loss.
- b_i is not the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on H:

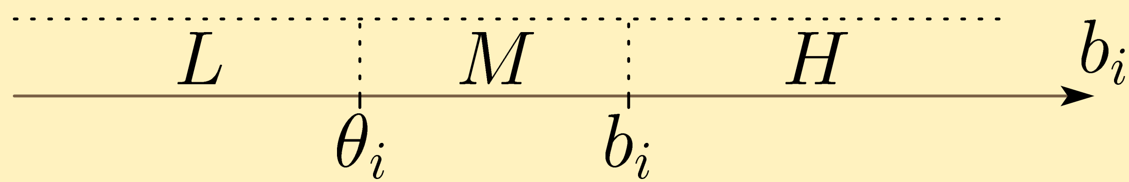
Independent Private Values

Second-price auction Player i sets his bid, b_i.

$$v_i(b_i; b_{-i}; \theta_i) = \begin{cases} \theta_i - b_j^* & \text{if } b_i > b_j \text{ for all } j \neq i \\ & \quad \text{and } b_j^* \equiv \max_{j \neq i} b_j, \\ 0 & \text{if } b_i \leq b_j \text{ for some } j \neq i. \end{cases}$$

Proposition 13.1 $s_i(\theta_i) = \theta_i$ for all $i \in N$.

$(b_i > \theta)$



- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ on L:
- b_i is the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on M:
- b_i is not the highest, and $b_j^* \equiv \max_{j \neq i} b_j$ is on H: He loses. Bidding θ_i leads to the same result.

Bidding θ_i weakly dominates any lower bid.

13.1.3 First-price sealed-bid auctions

First-price auction Player i sets his bid, b_i. The highest bidder wins, and he pays a price equal to *his bid*.

Claim 13.1 Bidding his valuation is a dominated strategy.

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 The higher a player's valuation, the higher is his bid. That is, if $\theta'_j > \theta''_j$, then $s_j(\theta'_j) > s_j(\theta''_j)$.

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta_j' > \theta_j''$, then $s_j(\theta_j') > s_j(\theta_j'')$.

The prob. that i's bid is higher than j's bid is

$$\Pr\{s_j(\theta_j) < b_i\} = \Pr\{\theta_j < s_j^{-1}(b_i)\} = F_j(s_j^{-1}(b_i)).$$

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta'_j > \theta''_j$, then $s_j(\theta'_j) > s_j(\theta''_j)$.

The prob. that i's bid is higher than j's bid is

$$\Pr\{s_j(\theta_j) < b_i\} = \Pr\{\theta_j < s_j^{-1}(b_i)\} = F_j(s_j^{-1}(b_i)).$$

By the IPV setting, i's expected payoff is

$$E_{\theta_{-i}}[v_i(b_i; s_{-i}(\theta_{-i}); \theta_i)|\theta_i] = \underbrace{\prod_{j \neq i} \left[F_j(s_j^{-1}(b_i))\right]}_{\text{Prob. of the win}} \times (\theta_i - b_i).$$

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta'_j > \theta''_j$, then $s_j(\theta'_j) > s_j(\theta''_j)$.

The prob. that i's bid is higher than j's bid is

$$\Pr\{s_j(\theta_j) < b_i\} = \Pr\{\theta_j < s_j^{-1}(b_i)\} = F_j(s_j^{-1}(b_i)).$$

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$$E_{\theta_{-i}}[v_i(b_i; s_{-i}(\theta_{-i}); \theta_i)|\theta_i] = \prod_{j \neq i} \left[F_j(s_j^{-1}(b_i))\right] \times (\theta_i - b_i).$$

Player i's type, θ_i, is drawn from the same interval $\theta_i \in [\underline{\theta}, \bar{\theta}]$, the same cdf $F(\cdot)$, and $\underline{\theta} \geq 0$.

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta'_j > \theta''_j$, then $s_j(\theta'_j) > s_j(\theta''_j)$.

The prob. that i's bid is higher than j's bid is

$$\Pr\{s_j(\theta_j) < b_i\} = \Pr\{\theta_j < s_j^{-1}(b_i)\} = F_j(s_j^{-1}(b_i)).$$

By the IPV setting, i's expected payoff is

$$E_{\theta_{-i}}[v_i(b_i; s_{-i}(\theta_{-i}); \theta_i)|\theta_i] = \prod_{j \neq i} \left[F_j(s_j^{-1}(b_i))\right] \times (\theta_i - b_i).$$

Consider a symmetric equilibrium. Given that all other players employ $s(\cdot)$, i's maximization problem is

$$\max_{b \geq 0} \left[F(s^{-1}(b))\right]^{n-1} (\theta_i - b).$$

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta_j' > \theta_j''$, then $s_j(\theta_j') > s_j(\theta_j'')$.

Consider a symmetric equilibrium. Given that all other players employ $s(\cdot)$, i's maximization problem is

$$\max_{b \geq 0} \left[F(s^{-1}(b))\right]^{n-1} (\theta_i - b).$$

The first-order condition of the problem is

$$-\left[F(s^{-1}(b))\right]^{n-1}$$
$$+(n-1)\left[F(s^{-1}(b))\right]^{n-2} f(s^{-1}(b)) \frac{ds^{-1}(b)}{db}(\theta_i - b) = 0.$$

Note that $\dfrac{ds^{-1}(b)}{db} = \dfrac{1}{s'(s^{-1}(b))}$.

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta_j' > \theta_j''$, then $s_j(\theta_j') > s_j(\theta_j'')$.

Consider a symmetric equilibrium.

The first-order condition of the problem becomes

$$- [F(\theta)]^{n-1} \\ +(n-1)[F(\theta)]^{n-2} f(\theta) \frac{1}{s'(\theta)} (\theta - s(\theta)) = 0.$$

We use the fact that $b = s(\theta)$ and $s^{-1}(b) = \theta$.

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta'_j > \theta''_j$, then $s_j(\theta'_j) > s_j(\theta''_j)$.

The first-order condition of the problem becomes

$$-\left[F(\theta)\right]^{n-1} + (n-1)\left[F(\theta)\right]^{n-2} f(\theta)\frac{1}{s'(\theta)}(\theta - s(\theta)) = 0.$$

This can be rewritten as

$$\begin{aligned} & \left[F(\theta)\right]^{n-1} s'(\theta) + (n-1)\left[F(\theta)\right]^{n-2} f(\theta)s(\theta) \\ = \; & (n-1)\left[F(\theta)\right]^{n-2} f(\theta)\theta. \end{aligned} \quad (1)$$

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta'_j > \theta''_j$, then $s_j(\theta'_j) > s_j(\theta''_j)$.

This can be rewritten as

$$\begin{aligned} & [F(\theta)]^{n-1} s'(\theta) + (n-1)[F(\theta)]^{n-2} f(\theta)s(\theta) \\ = \ & (n-1)[F(\theta)]^{n-2} f(\theta)\theta. \end{aligned} \tag{1}$$

By defining $g(\theta) \equiv [F(\theta)]^{n-1}$, we can use the fact that

$$(g(\theta)s(\theta))' = g(\theta)s'(\theta) + g'(\theta)s(\theta), \quad (A)$$
$$(g(\theta)\theta)' = g(\theta) + g'(\theta)\theta. \quad (B)$$

The RHS in (A) is the same with the LHS in (1).
From (B), the RHS in (1) is $([F(\theta)]^{n-1}\theta)' - [F(\theta)]^{n-1}$.

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta_j' > \theta_j''$, then $s_j(\theta_j') > s_j(\theta_j'')$.

This can be rewritten as

$$\begin{aligned} & [F(\theta)]^{n-1} s'(\theta) + (n-1)[F(\theta)]^{n-2} f(\theta)s(\theta) \\ = \ & (n-1)[F(\theta)]^{n-2} f(\theta)\theta. \end{aligned} \tag{1}$$

By defining $g(\theta) \equiv [F(\theta)]^{n-1}$, we can use the fact that

$$(g(\theta)s(\theta))' = g(\theta)s'(\theta) + g'(\theta)s(\theta), \quad (A)$$
$$(g(\theta)\theta)' = g(\theta) + g'(\theta)\theta. \quad (B)$$

(1) becomes

$$([F(\theta)]^{n-1} s(\theta))' = ([F(\theta)]^{n-1} \theta)' - [F(\theta)]^{n-1}$$

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta_j' > \theta_j''$, then $s_j(\theta_j') > s_j(\theta_j'')$.

The condition for a symmetric Bayesian Nash equilibrium

$$([F(\theta)]^{n-1}\, s(\theta))' = ([F(\theta)]^{n-1}\, \theta)' - [F(\theta)]^{n-1}$$

Using the above equation, we obtain

$$\begin{aligned}&\int_{\underline{\theta}}^{\theta} ([F(x)]^{n-1}\, s(x))' dx \\ &= \int_{\underline{\theta}}^{\theta} ([F(\theta)]^{n-1}\, \theta)' dx - \int_{\underline{\theta}}^{\theta} [F(\theta)]^{n-1}\, dx.\end{aligned}$$

The LHS is the LHS of (13.6) in Tadelis ($\because F(\underline{\theta}) = 0$).

Independent Private Values

First-price auction Player i sets his bid, b_i.

Assumption 13.1 If $\theta_j' > \theta_j''$, then $s_j(\theta_j') > s_j(\theta_j'')$.

The condition for a symmetric Bayesian Nash equilibrium

$$([F(\theta)]^{n-1} s(\theta))' = ([F(\theta)]^{n-1} \theta)' - [F(\theta)]^{n-1}$$

Using the above equation, we obtain

$$\int_{\underline{\theta}}^{\theta} ([F(x)]^{n-1} s(x))' dx = \int_{\underline{\theta}}^{\theta} ([F(\theta)]^{n-1} \theta)' dx - \int_{\underline{\theta}}^{\theta} [F(\theta)]^{n-1} dx.$$

Solving (13.6) with respect to $s(\theta)$, we obtain (13.7).

13.2 Common Values Case

Common values case Consider a second-price auction. There are two bidders. A product's value, v, is common among them although each bidder receives only the signal of the value, $\theta_i = \{L, H\}$, where L and H respectively represent low- and high-quality.

Common Values

Common values case Consider a second-price auction. There are two bidders. A product's value, v, is common among them although each bidder receives only the signal of the value, $\theta_i = \{L, H\}$.

The realization of v depends on the following probability distribution:

$$\Pr\{v = 10\} = \Pr\{v = 30\} = 1/4, \quad \Pr\{v = 20\} = 1/2.$$

The relation between the signals and the true value is

1. If $v = 10$, $\theta_1 = \theta_2 = L$.
2. If $v = 30$, $\theta_1 = \theta_2 = H$.
3. If $v = 20$, $\theta_i = L$ and $\theta_j = H$ $(i, j = 1, 2,\ i \neq j)$.

Common Values

Common values case Consider a second-price auction.

There are two bidders.

The realization of v

$$\Pr\{v = 10\} = \Pr\{v = 30\} = 1/4, \quad \Pr\{v = 20\} = 1/2.$$

The relation between the signals and the true value is

		θ_2: L	θ_2: H
θ_1	L	1/4	1/4
	H	1/4	1/4

Ex post belief $(i, j = 1, 2,\ i \neq j)$

$$\begin{aligned} & \Pr\{\theta_j = L \mid \theta_i = L\} \\ = \ & \Pr\{\theta_j = H \mid \theta_i = L\} = 1/2, \\ & \Pr\{\theta_j = L \mid \theta_i = H\} \\ = \ & \Pr\{\theta_j = H \mid \theta_i = H\} = 1/2. \end{aligned}$$

Common Values

Common values case Consider a second-price auction.

There are two bidders.

The realization of v

$$\Pr\{v=10\}=\Pr\{v=30\}=1/4, \quad \Pr\{v=20\}=1/2.$$

The relation between the signals and the true value is

		θ_2	
		L	H
θ_1	L	1/4	1/4
	H	1/4	1/4

Ex post belief $(i, j = 1, 2,\ i \neq j)$

$$\begin{aligned} & \Pr\{\theta_j = L \mid \theta_i = L\} \\ = & \Pr\{\theta_j = H \mid \theta_i = L\} = 1/2, \\ & \Pr\{\theta_j = L \mid \theta_i = H\} \\ = & \Pr\{\theta_j = H \mid \theta_i = H\} = 1/2. \end{aligned}$$

There is a symmetric Bayesian Nash equilibrium.

Common Values

Realization of v The relation between θ_i and v is

If $v = 10$, $\theta_1 = \theta_2 = L$; If $v = 30$, $\theta_1 = \theta_2 = H$;

If $v = 20$, $\theta_i = L$ and $\theta_j = H$ $(i, j = 1, 2,\ i \neq j)$.

$s_j(\cdot)$: Bid $b_j(k)$ if $\theta_j = k$ $(k = L, H)$ and $b_j(L) \leq b_j(H)$.

Bid $(\theta_i = L)$	$E[v_i(b_i, s_j; L) \mid L]$
$b_i(L) \in [0, b_j(L))$	$0,$
$b_i(L) = b_j(L)$	$\frac{1}{2}\frac{(10 - b_j(L))}{2},$
$b_i(L) \in (b_j(L), b_j(H))$	$\frac{1}{2}(10 - b_j(L)),$
$b_i(L) = b_j(H)$	$\frac{1}{2}(10 - b_j(L)) + \frac{1}{2}\frac{(20 - b_j(H))}{2},$
$b_i(L) \in (b_j(H), \infty)$	$\frac{1}{2}(10 - b_j(L)) + \frac{1}{2}(20 - b_j(H)).$

Common Values

Realization of v The relation between θ_i and v is

If $v = 10$, $\theta_1 = \theta_2 = L$; If $v = 30$, $\theta_1 = \theta_2 = H$;

If $v = 20$, $\theta_i = L$ and $\theta_j = H$ $(i, j = 1, 2,\ i \neq j)$.

$s_j(\cdot)$: Bid $b_j(k)$ if $\theta_j = k$ $(k = L, H)$ and $b_j(L) \leq b_j(H)$.

Bid $(\theta_i = L)$	$E[v_i(b_i, s_j; L)\|L]$
$b_i(L) \in [0, b_j(L))$	0,
$b_i(L) = b_j(L)$	$\frac{1}{2}\frac{(10 - b_j(L))}{2}$,
$b_i(L) \in (b_j(L), b_j(H))$	$\frac{1}{2}(10 - b_j(L))$,
$b_i(L) = b_j(H)$	$\frac{1}{2}(10 - b_j(L)) + \frac{1}{2}\frac{(20 - b_j(H))}{2}$,
$b_i(L) \in (b_j(H), \infty)$	$\frac{1}{2}(10 - b_j(L)) + \frac{1}{2}(20 - b_j(H))$.

What is the cond. player i with $\theta_i = L$ bids $b_i(L) = b_j(L)$?

Common Values

Realization of v The relation between θ_i and v is

If $v = 10$, $\theta_1 = \theta_2 = L$; If $v = 30$, $\theta_1 = \theta_2 = H$;

If $v = 20$, $\theta_i = L$ and $\theta_j = H$ $(i, j = 1, 2,\ i \neq j)$.

$s_j(\cdot)$: Bid $b_j(k)$ if $\theta_j = k$ $(k = L, H)$ and $b_j(L) \leq b_j(H)$.

Bid $(\theta_i = H)$	$E[v_i(b_i, s_j; H) \vert H]$
$b_i(H) \in [0, b_j(L))$	0,
$b_i(H) = b_j(L)$	$\frac{1}{2}\frac{(20 - b_j(L))}{2}$,
$b_i(H) \in (b_j(L), b_j(H))$	$\frac{1}{2}(20 - b_j(L))$,
$b_i(H) = b_j(H)$	$\frac{1}{2}(20 - b_j(L)) + \frac{1}{2}\frac{(30 - b_j(H))}{2}$,
$b_i(H) \in (b_j(H), \infty)$	$\frac{1}{2}(20 - b_j(L)) + \frac{1}{2}(30 - b_j(H))$.

Common Values

Realization of v The relation between θ_i and v is

If $v = 10$, $\theta_1 = \theta_2 = L$; If $v = 30$, $\theta_1 = \theta_2 = H$;

If $v = 20$, $\theta_i = L$ and $\theta_j = H$ $(i, j = 1, 2,\ i \neq j)$.

$s_j(\cdot)$: Bid $b_j(k)$ if $\theta_j = k$ $(k = L, H)$ and $b_j(L) \leq b_j(H)$.

Bid $(\theta_i = H)$	$E[v_i(b_i, s_j; H) \mid H]$
$b_i(H) \in [0, b_j(L))$	0,
$b_i(H) = b_j(L)$	$\frac{1}{2}\frac{(20 - b_j(L))}{2}$,
$b_i(H) \in (b_j(L), b_j(H))$	$\frac{1}{2}(20 - b_j(L))$,
$b_i(H) = b_j(H)$	$\frac{1}{2}(20 - b_j(L)) + \frac{1}{2}\frac{(30 - b_j(H))}{2}$,
$b_i(H) \in (b_j(H), \infty)$	$\frac{1}{2}(20 - b_j(L)) + \frac{1}{2}(30 - b_j(H))$.

What is the cond. player i with $\theta_i = H$ bids $b_i(H) = b_j(H)$?