Chapter §9.C in MWG: Beliefs and Sequential Rationality
Chapter 15 in Tadelis: Sequential Rationality with Incomplete Information

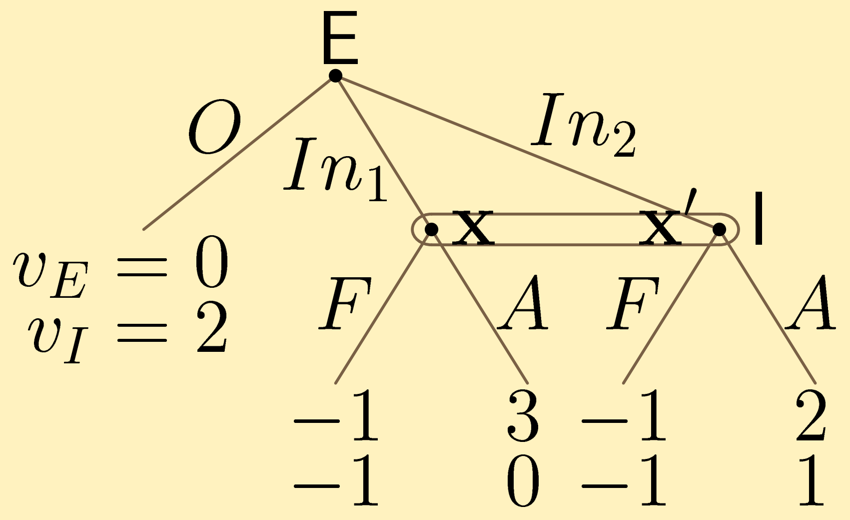
- The problem with subgame perfection
- Perfect Bayesian Equilibrium
- Sequential Equilibrium

15.1 The problem with subgame perfection

§9.C Beliefs and Sequential Rationality

Example 9.C.1 No subgame except for the whole game.

E/I	F	A
O	$\mathbf{0, 2}$	$0, 2$
In_1	$-1, -1$	$\mathbf{3, 0}$
In_2	$-1, -1$	$2, 1$

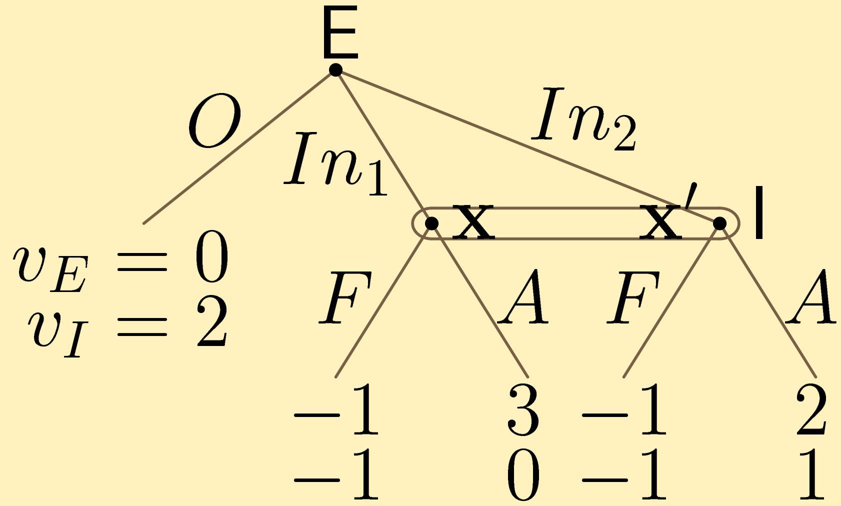


There are two subgame perfect Nash equilibria.

§9.C Beliefs and Sequential Rationality

Example 9.C.1 No subgame except for the whole game.

E/I	F	A
O	$\mathbf{0, 2}$	$0, 2$
In_1	$-1, -1$	$\mathbf{3, 0}$
In_2	$-1, -1$	$2, 1$

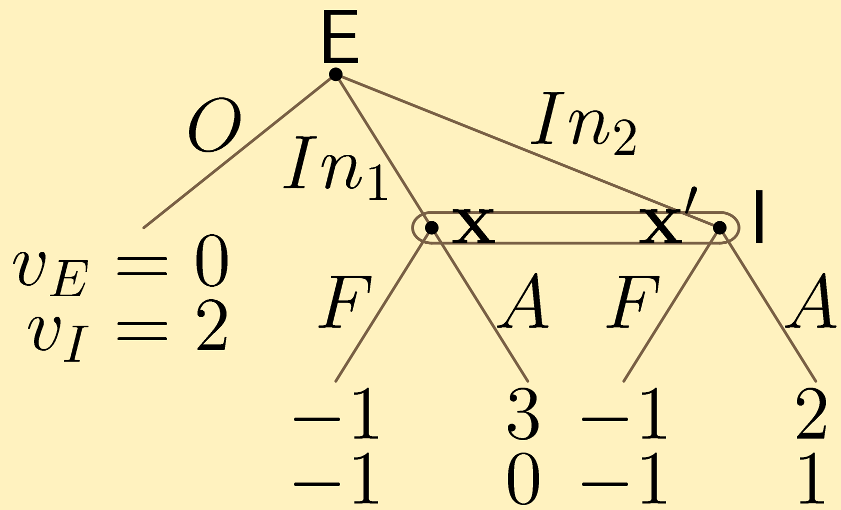


There are two subgame perfect Nash equilibria. However, once player E enters, whether plays In_1 or In_2, it is optimal for player I to play A.

§9.C Beliefs and Sequential Rationality

Example 9.C.1 No subgame except for the whole game.

E/I	F	A
O	$\mathbf{0, 2}$	$0, 2$
In_1	$-1, -1$	$\mathbf{3, 0}$
In_2	$-1, -1$	$2, 1$

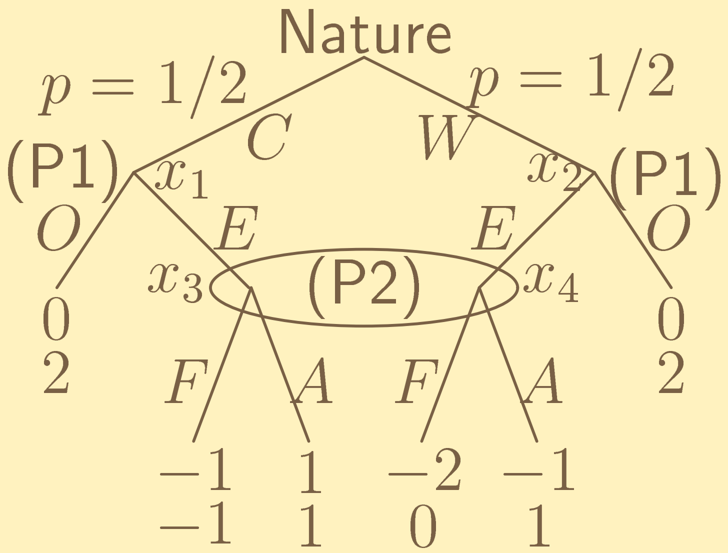


There are two subgame perfect Nash equilibria. However, once player E enters, whether plays In_1 or In_2, it is optimal for player I to play A. Thus, (O, F) is NOT consistent with the spirit of sequential rationality.

§9.C Beliefs and Sequential Rationality

Figure 15.2 No subgame except for the whole game.

P1/P2	F	A
OO	$0, 2$	$0, 2$
OE	$-1, 1$	$-\frac{1}{2}, \frac{3}{2}$
EO	$-\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{3}{2}$
EE	$-\frac{3}{2}, -\frac{1}{2}$	$0, 1$

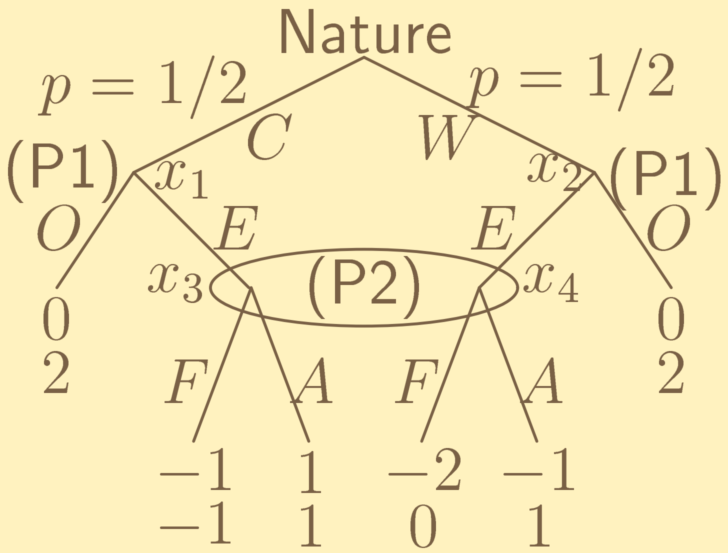


There are two subgame perfect Nash equilibria.

§9.C Beliefs and Sequential Rationality

Figure 15.2 No subgame except for the whole game.

P1/P2	F	A
OO	$0, 2$	$0, 2$
OE	$-1, 1$	$-\frac{1}{2}, \frac{3}{2}$
EO	$-\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{3}{2}$
EE	$-\frac{3}{2}, -\frac{1}{2}$	$0, 1$



There are two subgame perfect Nash equilibria. However, once P1 enters (E), it is optimal for P2 to play A irrespective of Nature's choice.

15.2 Perfect Bayesian Equilibrium

Definition 15.1 Let $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ be a Bayesian Nash equilibrium profile of strategies in a game of incomplete information.

An information set is **on the equilibrium path** if given σ^* and the distribution of types, it is reached *with positive probability*.

An information set is **off the equilibrium path** if given σ^* and the distribution of types, it is reached *with zero probability*.

Definition 15.2 A **system of beliefs** μ of an extensive-form game assigns a probability distribution over decision nodes to every information set.
That is, for every information set $h \in H$ and every decision node $x \in h$, $\mu(x) \in [0, 1]$ is the probability that player i who moves in information set h assigns to his being at x, where

$$\sum_{x \in h} \mu(x) = 1,$$

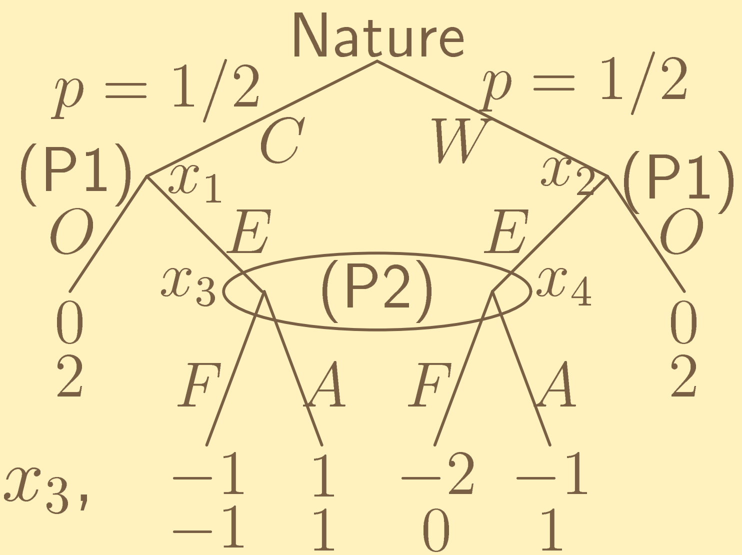
for every information set $h \in H$.

Definition 15.2

That is, for every information set $h \in H$ and every decision node $x \in h$, $\mu(x) \in [0,1]$ is the probability that player i who moves in information set h assigns to his being at x, where

$$\sum_{x \in h} \mu(x) = 1,$$

for every information set $h \in H$.



Player 2's belief $(\mu(x_4) = 1 - \mu(x_3))$

$\mu(x_3)$: Player 2's belief that he is at x_3,

$\mu(x_4)$: Player 2's belief that he is at x_4.

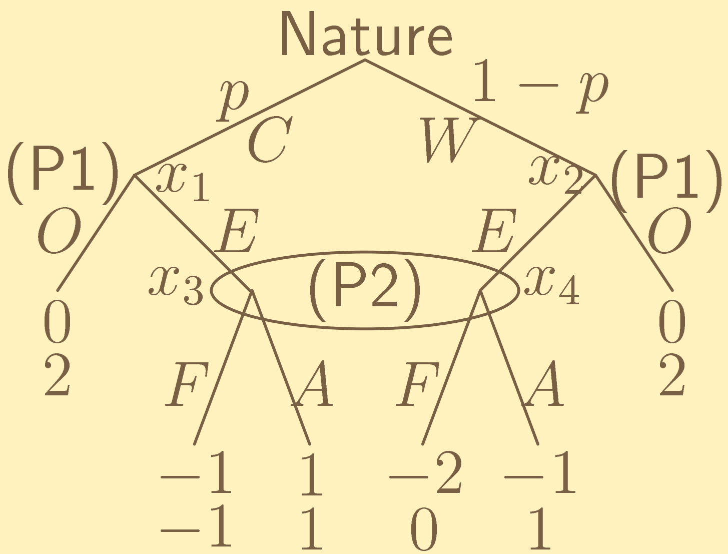
Beliefs and Sequential Rationality

Requirement 15.1 Every player will have a well-defined belief over where he is in each of his information sets. That is, the game will have a *system of beliefs*.

How do we determine the beliefs?

Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is EO, $\mu(x_3) = 1$.



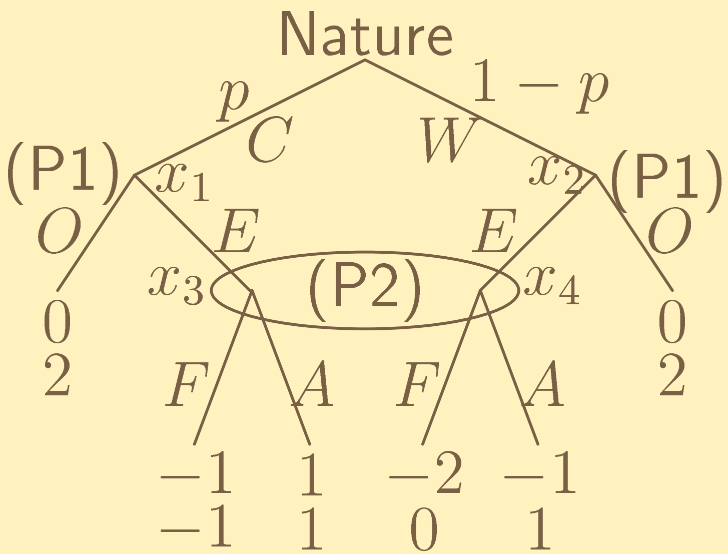
Beliefs and Sequential Rationality

Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is EO, $\mu(x_3) = 1$.

If P1's strategy is (σ_C, σ_W), where σ_k is the probability that he plays E when he is k ($k = C, W$),

$$\mu(x_3) = \frac{p\sigma_C}{p\sigma_C + (1-p)\sigma_W}.$$

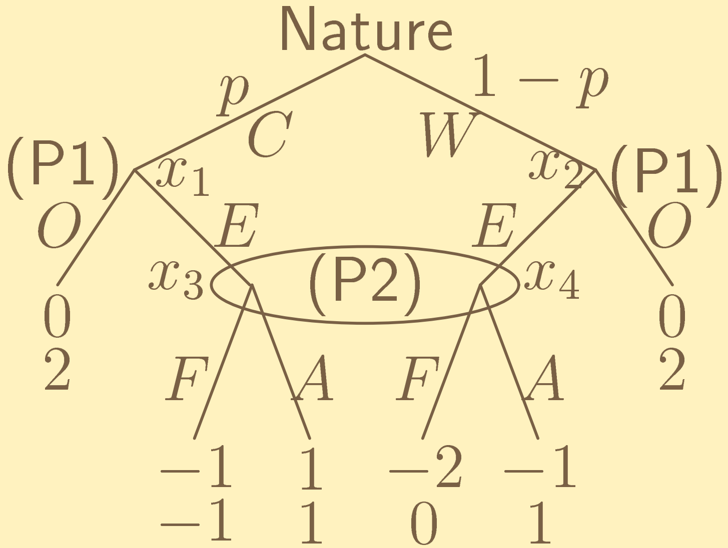


Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is EO, $\mu(x_3) = 1$.

If P1's strategy is (σ_C, σ_W), where σ_k is the probability that he plays E when he is k $(k = C, W)$,

$$\mu(x_3) = \frac{p\sigma_C}{p\sigma_C + (1-p)\sigma_W}.$$



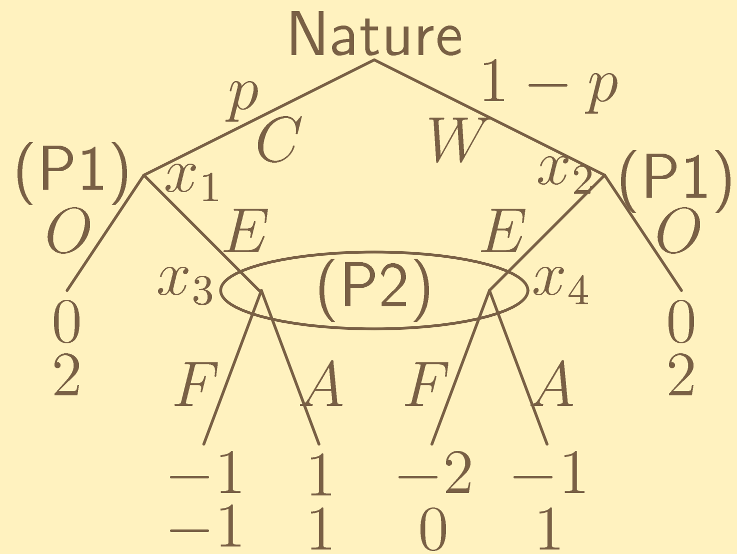
Requirement 15.2 Let $\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*)$ be a Bayesian Nash equilibrium profile of strategies. We require that in all information sets beliefs that are on the equilibrium path be consistent with *Bayes' rule*.

Beliefs and Sequential Rationality

Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is (σ_C, σ_W), where σ_k is the probability that he plays E when he is k $(k = C, W)$,

$$\mu(x_3) = \frac{p\sigma_C}{p\sigma_C + (1-p)\sigma_W}.$$



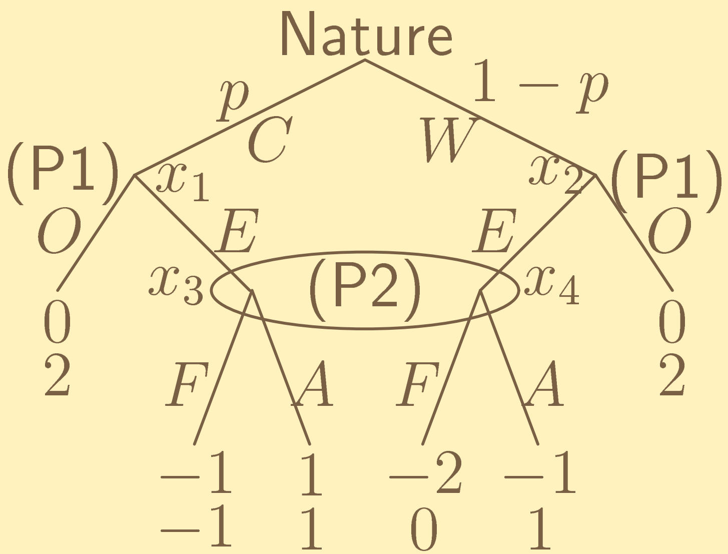
If P1's strategy is $(\sigma_C, \sigma_W) = (0, 0)$ (that is, OO), the information set of P2 is off the equilibrium path.

Beliefs and Sequential Rationality

Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is (σ_C, σ_W), where σ_k is the probability that he plays E when he is k $(k = C, W)$,

$$\mu(x_3) = \frac{p\sigma_C}{p\sigma_C + (1-p)\sigma_W}.$$



If P1's strategy is $(\sigma_C, \sigma_W) = (0, 0)$ (that is, OO), the information set of P2 is off the equilibrium path.

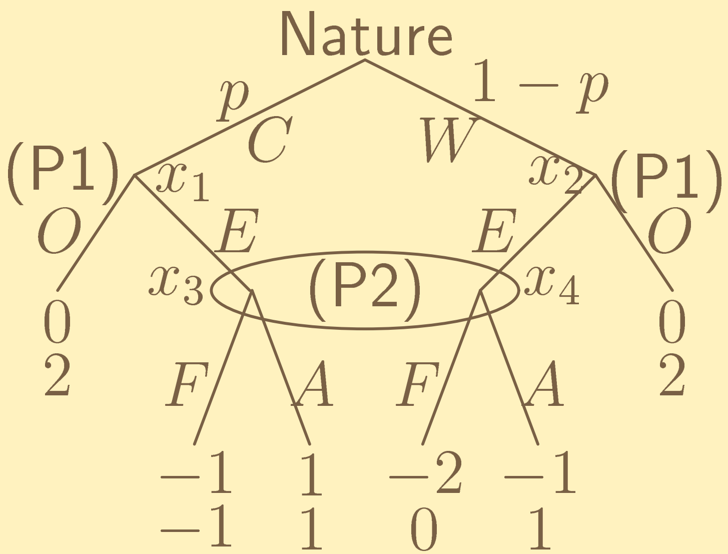
Requirement 15.3 At information sets that are off the equilibrium path, any belief can be assigned.

Beliefs and Sequential Rationality

Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is (σ_C, σ_W), where σ_k is the probability that he plays E when he is k $(k = C, W)$,

$$\mu(x_3) = \frac{p\sigma_C}{p\sigma_C + (1-p)\sigma_W}.$$



Requirement 15.2 In all information sets beliefs that are on the equilibrium path be consistent with *Bayes' rule*.

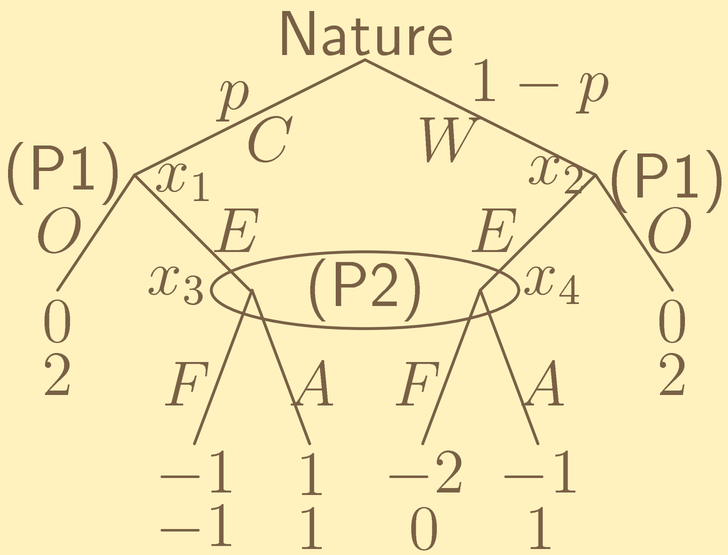
Requirement 15.3 At information sets that are off the equilibrium path, any belief can be assigned.

Beliefs and Sequential Rationality

Requirement 15.1 The game will have a *system of beliefs*.

If P1's strategy is (σ_C, σ_W), where σ_k is the probability that he plays E when he is k $(k = C, W)$,

$$\mu(x_3) = \frac{p\sigma_C}{p\sigma_C + (1-p)\sigma_W}.$$



Requirement 15.2 In all information sets beliefs that are on the equilibrium path be consistent with *Bayes' rule*.

Requirement 15.4 Given their beliefs, players' strategies must be sequentially rational. That is, in every information set, players will play a best response to their beliefs.

Beliefs and Sequential Rationality

Def. Expected utility $E[v_i(\sigma_i, \sigma_{-i}, \theta_i)|h, \mu]$

Player i's expected payoff starting at his information set h if the belief is given by μ, if player i follows σ_i and others follow σ_{-i}, and if his type is θ_i.

$\iota(h)$: the player who moves at information set H.

Def. 9.C.2 A strategy profile σ of an extensive-form game is **sequentially rational** at information set h given a system of beliefs μ if $\forall \hat{\sigma}_{\iota(h)} \in \Delta(S_{\iota(h)})$,

$$\begin{aligned} & E[v_{\iota(h)}(\sigma_{\iota(h)}, \sigma_{-\iota(h)}, \theta_i)|h, \mu] \\ \geq\ & E[v_{\iota(h)}(\hat{\sigma}_{\iota(h)}, \sigma_{-\iota(h)}, \theta_i)|h, \mu]. \end{aligned}$$

Beliefs and Sequential Rationality

Def. Expected utility $E[v_i(\sigma_i, \sigma_{-i}, \theta_i)|h, \mu]$

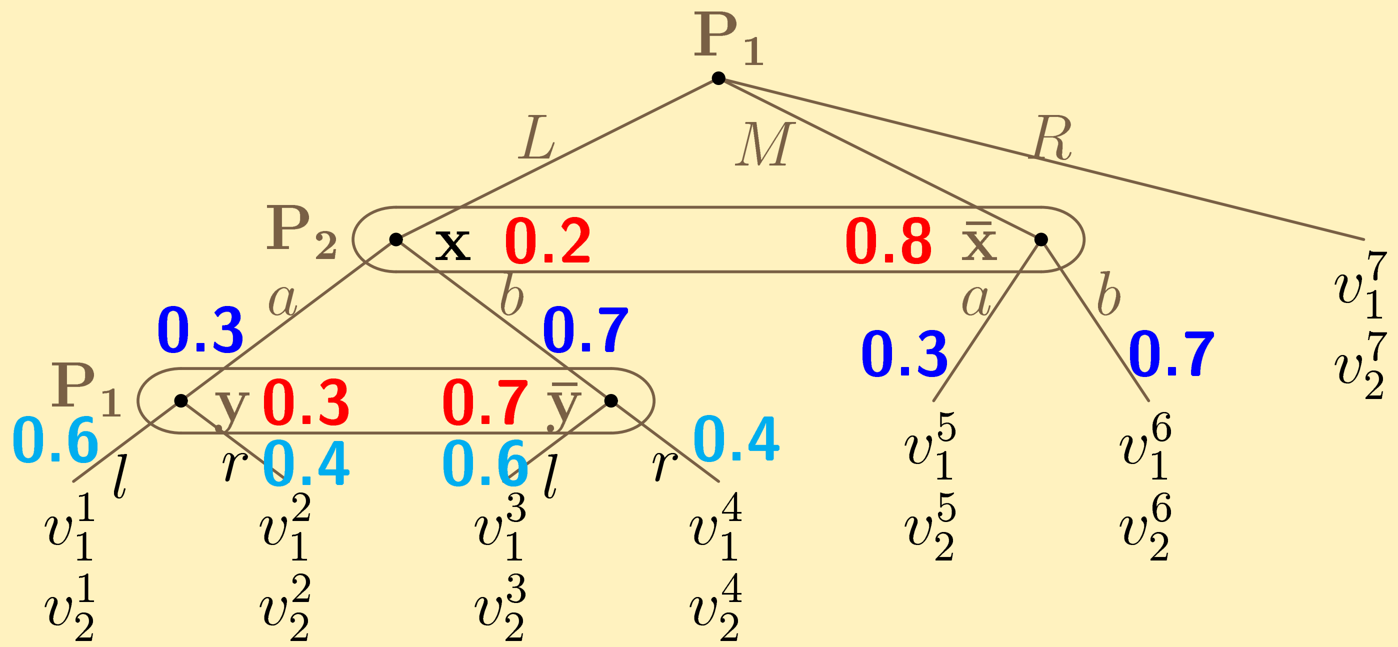
$\iota(h)$: the player who moves at information set H.

Def. 9.C.2 A strategy profile σ of an extensive-form game is **sequentially rational** at information set h given a system of beliefs μ if $\forall \hat{\sigma}_{\iota(h)} \in \Delta(S_{\iota(h)})$,

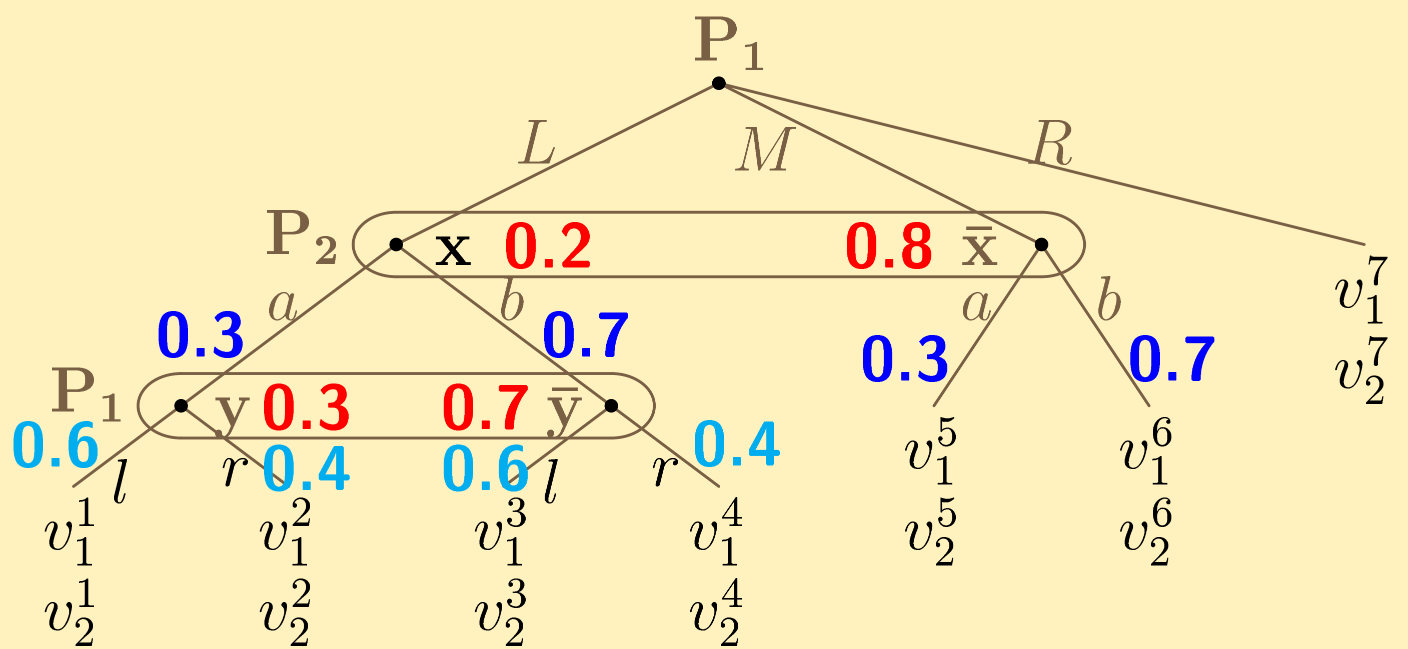
$$\begin{aligned} & E[v_{\iota(h)}(\sigma_{\iota(h)}, \sigma_{-\iota(h)}, \theta_i)|h, \mu] \\ \geq \ & E[v_{\iota(h)}(\hat{\sigma}_{\iota(h)}, \sigma_{-\iota(h)}, \theta_i)|h, \mu]. \end{aligned}$$

A strategy profile σ is sequentially rational given μ, if σ is sequentially rational at any information set h given μ.

Calculation of expected utility $E[v_i(\sigma_i, \sigma_{-i})|h, \mu]$



Calculation of expected utility $E[v_i(\sigma_i, \sigma_{-i})|h, \mu]$

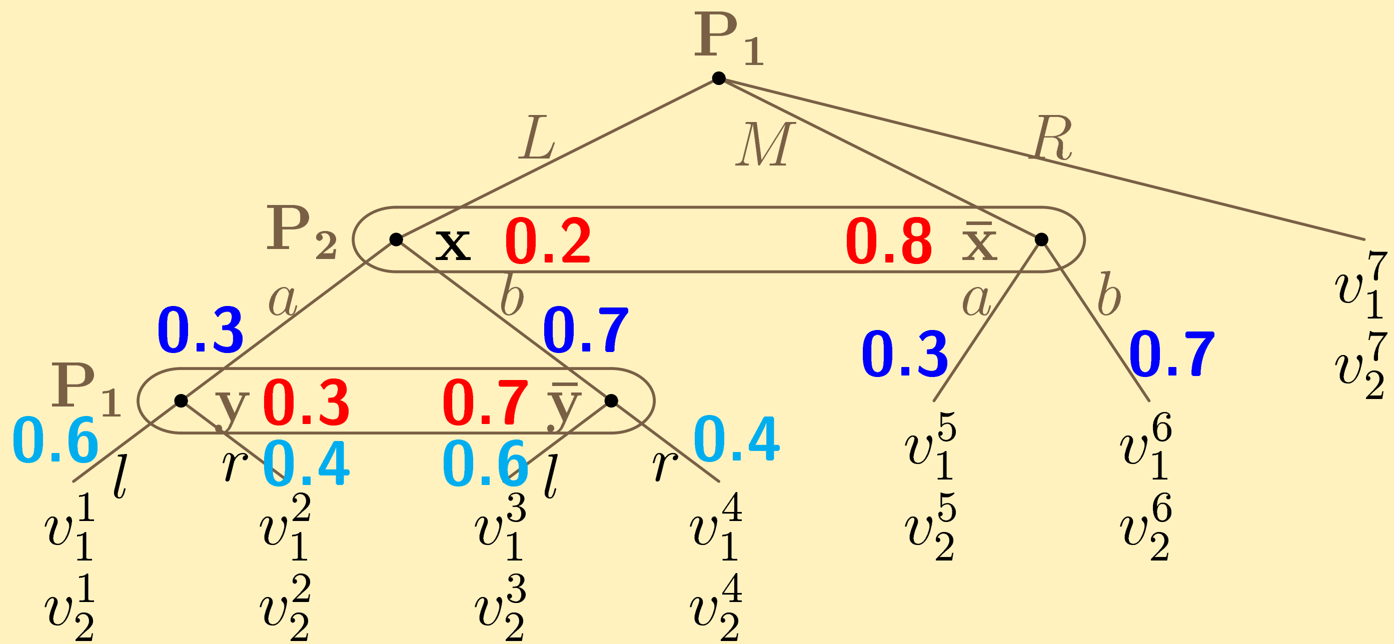


$\iota(h_x) = P_2$, $\iota(h_y) = P_1$.

$\mu(\mathbf{x}) = \mathbf{0.2}$, $\mu(\bar{\mathbf{x}}) = \mathbf{0.8}$, $\mu(\mathbf{y}) = \mathbf{0.3}$, $\mu(\bar{\mathbf{y}}) = \mathbf{0.7}$.

$\sigma_1(\mathbf{l}) = \mathbf{0.6}$, $\sigma_1(\mathbf{r}) = \mathbf{0.4}$, $\sigma_2(\mathbf{a}) = \mathbf{0.3}$, $\sigma_2(\mathbf{b}) = \mathbf{0.7}$.

Calculation of expected utility $E[v_i(\sigma_i, \sigma_{-i})|h, \mu]$



$$E[v_1(\sigma_1, \sigma_2)|h_y, \mu] = 0.6[0.3v_1^1 + 0.7v_1^3] + 0.4[0.3v_1^2 + 0.7v_1^4],$$

$$E[v_2(\sigma_1, \sigma_2)|h_x, \mu] = 0.3[0.2(0.6v_2^1 + 0.4v_2^2) + 0.8v_2^5] + 0.7[0.2(0.6v_2^3 + 0.4v_2^4) + 0.8v_2^6].$$

Update of Beliefs

$\Pr(x|\sigma)$: Prob. of reaching a node x given play of σ.

Update of Beliefs

$\Pr(x|\sigma)$**:** Prob. of reaching a node x given play of σ.

$\Pr(h|\sigma) = \sum_{x \in h} \Pr(x|\sigma)$**:** Prob. of reaching an information set h given play of σ.

Update of Beliefs

$\Pr(x|\sigma)$: Prob. of reaching a node x given play of σ.

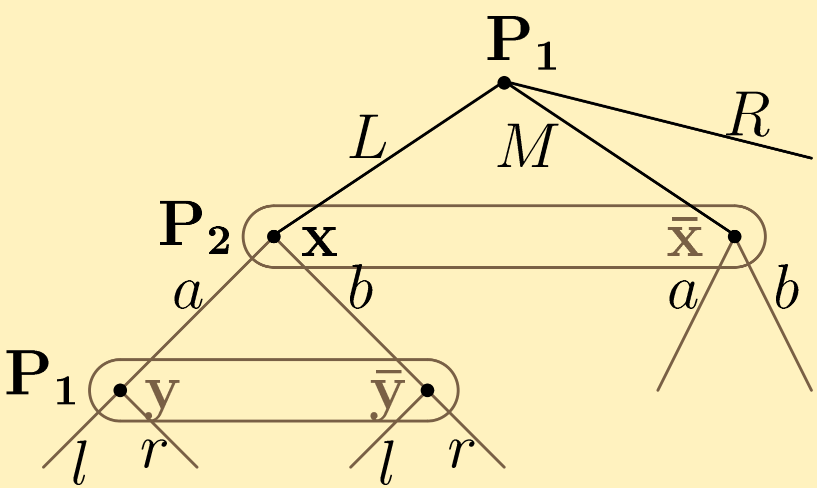
$\Pr(h|\sigma) = \sum_{x \in h} \Pr(x|\sigma)$: Prob. of reaching an information set h given play of σ.

$\Pr(x|h, \sigma) = \dfrac{\Pr(x|\sigma)}{\sum_{x' \in h} \Pr(x'|\sigma)}$: Cond. Prob. of being at x given that σ and the play reached h.

Update of Beliefs

$\Pr(x|h,\sigma) = \dfrac{\Pr(x|\sigma)}{\sum_{x' \in h} \Pr(x'|\sigma)}$: Cond. Prob. of being at x given that σ and the play reached h.

$$\begin{aligned}\Pr(y|h_y,\sigma) &= \frac{\Pr(y|\sigma)}{\Pr(h_y|\sigma)} \\ &= \frac{\sigma_1(L)\sigma_2(a)}{\sigma_1(L)\sigma_2(a)+\sigma_1(L)\sigma_2(b)} \\ &= \frac{\sigma_2(a)}{\sigma_2(a)+\sigma_2(b)}.\end{aligned}$$



(weak) Perfect Bayesian Equilibrium

Def. A system of beliefs μ is (weakly) consistent with σ if for all information set h with $\Pr(h|\sigma) > 0$ and all $x \in h$, $\mu(x) = \Pr(x|h, \sigma)$.

(weak) Perfect Bayesian Equilibrium

Def. A system of beliefs μ is (weakly) consistent with σ if for all information set h with $\Pr(h|\sigma) > 0$ and all $x \in h$, $\mu(x) = \Pr(x|h, \sigma)$.

Definition 15.3 A Bayesian Nash equilibrium profile σ^* together with a system of beliefs μ constitutes a (weak) perfect Bayesian equilibrium for an n-player game if

1. σ^* is sequentially rational given μ (Req. 15.4).
2. μ is weakly consistent with σ^* (Req. 15.1-3).

(weak) Perfect Bayesian Equilibrium

Definition 15.3 A Bayesian Nash equilibrium profile σ^* together with a system of beliefs μ constitutes a (weak) perfect Bayesian equilibrium for an n-player game if

1. σ^* is sequentially rational given μ (Req. 15.4).
2. μ is weakly consistent with σ^* (Req. 15.1-3).

Prop. 9.C.1 A strategy profile σ is a Nash equilibrium of Γ iff there is a system of beliefs μ such that

1. σ is sequentially rational given μ at all information sets h such that $\Pr(h|\sigma) > 0$,
2. μ is weakly consistent with σ.

(weak) Perfect Bayesian Equilibrium

Definition 15.3 A Bayesian Nash equilibrium profile σ^* together with a system of beliefs μ constitutes a (weak) perfect Bayesian equilibrium for an n-player game if

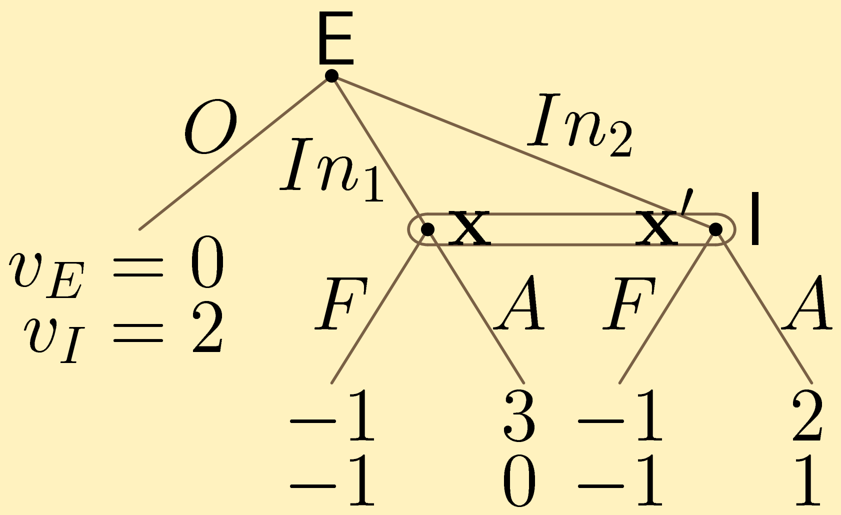
1. σ^* is sequentially rational given μ (Req. 15.4).
2. μ is weakly consistent with σ^* (Req. 15.1-3).

Prop. 15.1 If a strategy profile σ^* is a Bayesian Nash equilibrium of a Bayesian game Γ, and if σ^* induces all the information sets to be reached with positive probability, then σ^*, together with the belief system μ^* uniquely derived from σ^* and the distribution of types, constitutes a perfect Bayesian equilibrium for Γ.

Example

Example 9.C.1 Since A is a strictly dominant strategy, whatever μ is, only A is sequentially rational. Nash eq. (O, F) is NOT a weakly perfect Bayesian equilibrium.

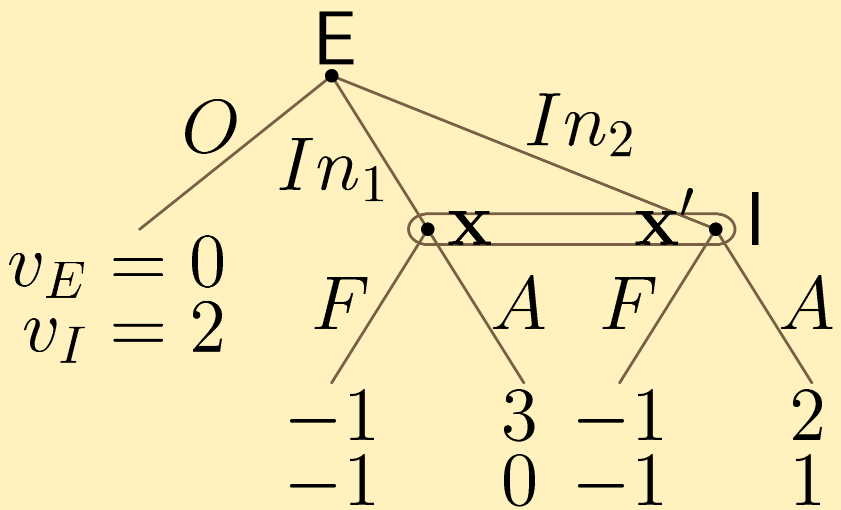
E/I	F	A
O	$\mathbf{0, 2}$	$0, 2$
In_1	$-1, -1$	$\mathbf{3, 0}$
In_2	$-1, -1$	$2, 1$



Example

Example 9.C.1 Since A is a strictly dominant strategy, whatever μ is, only A is sequentially rational. Nash eq. (O, F) is NOT a weakly perfect Bayesian equilibrium.

E/I	F	A
O	$\mathbf{0, 2}$	$0, 2$
In_1	$-1, -1$	$\mathbf{3, 0}$
In_2	$-1, -1$	$2, 1$

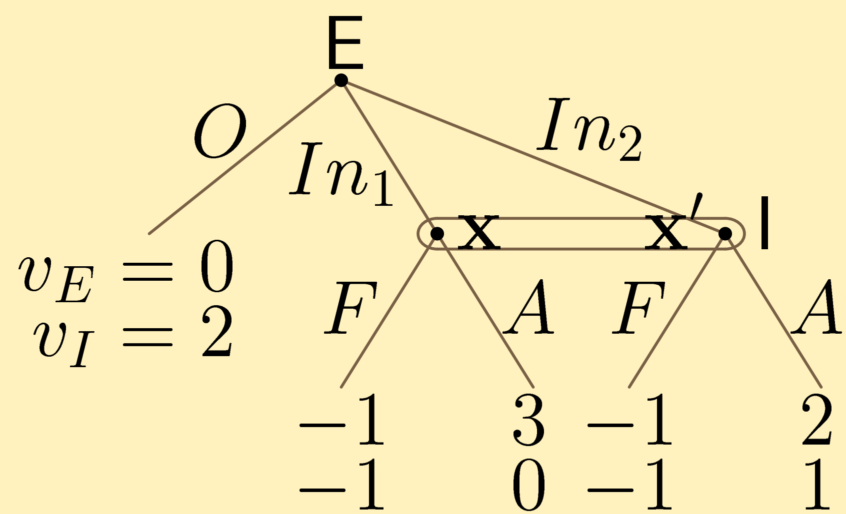


Given that E chooses $\sigma(O) = 1$, there is NO requirement for the belief of I.

Example

Example 9.C.1 Since A is a strictly dominant strategy, whatever μ is, only A is sequentially rational. Nash eq. (O, F) is NOT a weakly perfect Bayesian equilibrium.

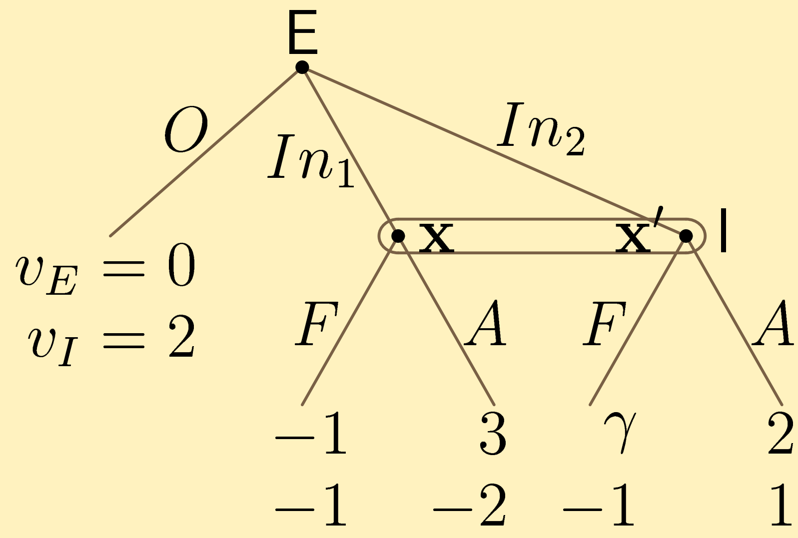
E/I	F	A
O	$\mathbf{0, 2}$	$0, 2$
In_1	$-1, -1$	$\mathbf{3, 0}$
In_2	$-1, -1$	$2, 1$



Given that E chooses $\sigma(O) = 1$, there is NO requirement for the belief of I. However, for any μ, A is a strictly dominant strategy.

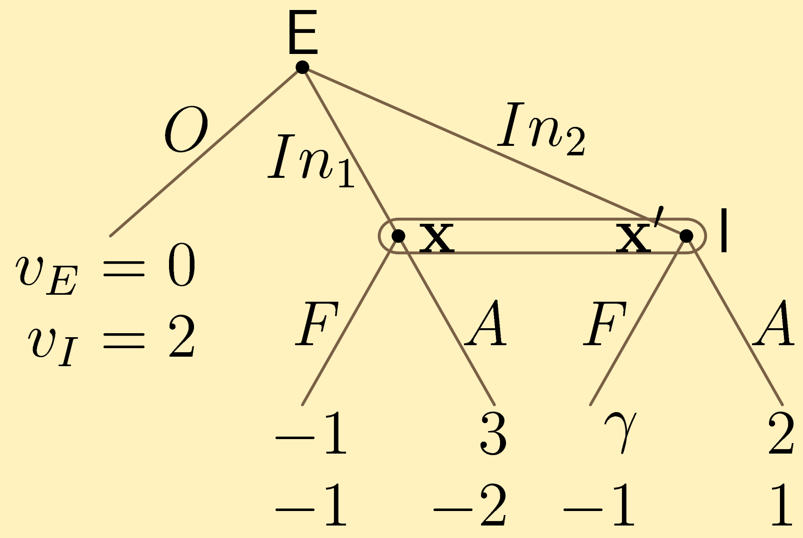
Example

Example 9.C.3 $(\gamma > -1, \gamma \neq 0)$



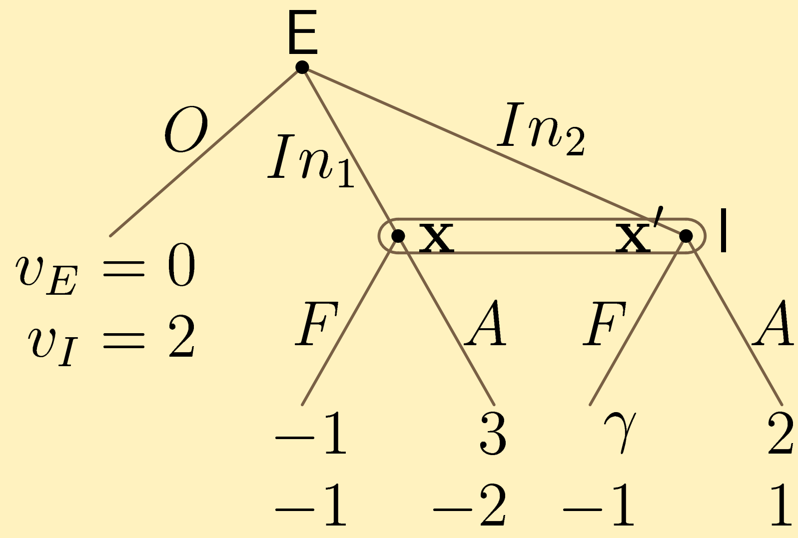
Example

Example 9.C.3 $(\gamma > -1,\ \gamma \neq 0)$



$$E[v_I(F,\sigma_E)|\mu] \gtreqless E[v_I(A,\sigma_E)|\mu] \Leftrightarrow \mu(\mathbf{x}) \gtreqless 2/3.$$

Example 9.C.3 $(\gamma > -1,\ \gamma \neq 0)$



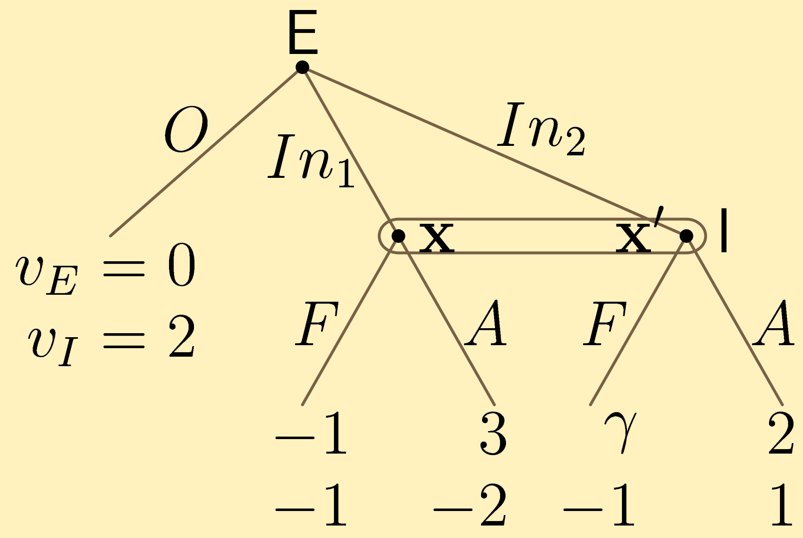
$$E[v_I(F,\sigma_E)|\mu] \gtreqless E[v_I(A,\sigma_E)|\mu] \Leftrightarrow \mu(\mathbf{x}) \gtreqless 2/3.$$

$(\gamma > 0)$:

$$\mu(\mathbf{x}) > 2/3 \Rightarrow \sigma_I(F) = 1 \Rightarrow \sigma_E(In_2) = 1 \Rightarrow \mu(\mathbf{x}) = 0.$$

Example

Example 9.C.3 $(\gamma > -1,\ \gamma \neq 0)$

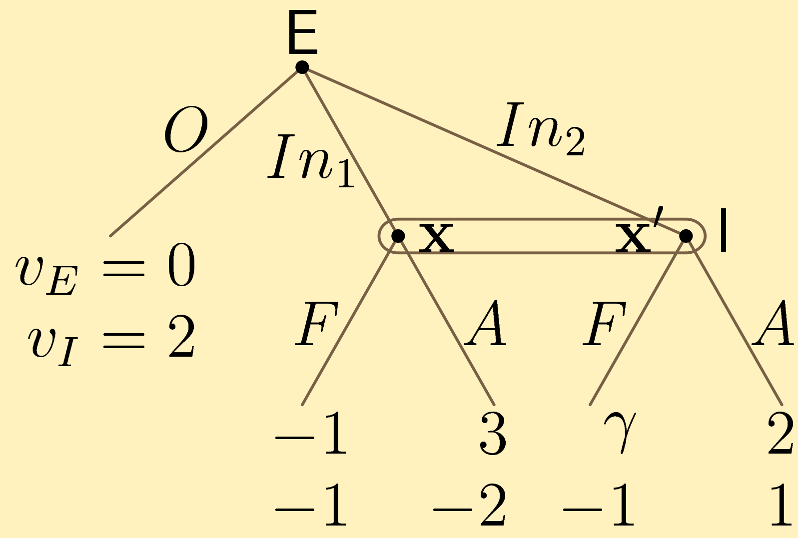


$$E[v_I(F, \sigma_E)|\mu] \gtreqless E[v_I(A, \sigma_E)|\mu] \Leftrightarrow \mu(\mathbf{x}) \gtreqless 2/3.$$

$(\gamma < 0)$:

$\mu(\mathbf{x}) > 2/3 \Rightarrow \sigma_I(F) = 1 \Rightarrow \sigma_E(In_1) = \sigma_E(In_2) = 0.$

Example 9.C.3 $(\gamma > -1, \gamma \neq 0)$



$E[v_I(F, \sigma_E)|\mu] \gtreqless E[v_I(A, \sigma_E)|\mu] \Leftrightarrow \mu(\mathbf{x}) \gtreqless 2/3.$

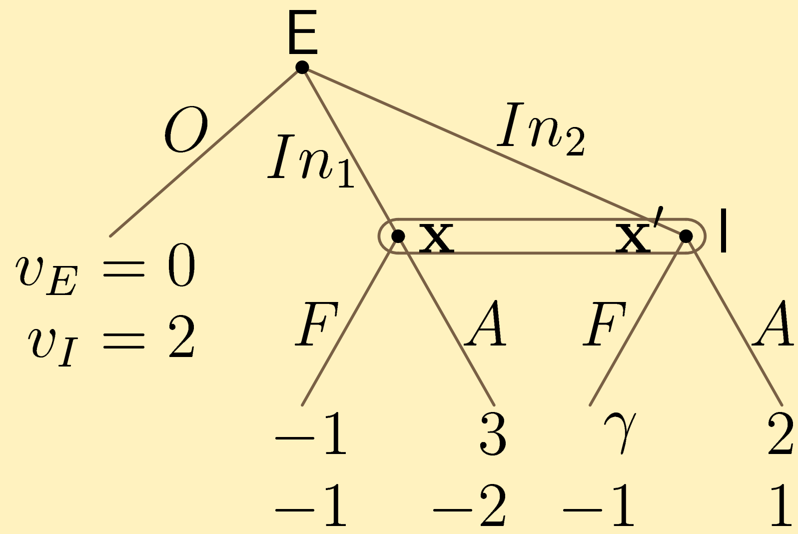
$(\gamma < 0)$:

$\mu(\mathbf{x}) > 2/3 \Rightarrow \sigma_I(F) = 1 \Rightarrow \sigma_E(In_1) = \sigma_E(In_2) = 0.$

$*$ If $\gamma < 0$, $\sigma_E(O) = 1$ and $\sigma_I(F) = 1$ with $\mu(\mathbf{x}) \geq 2/3$.

Example

Example 9.C.3 $(\gamma > -1,\, \gamma \neq 0)$



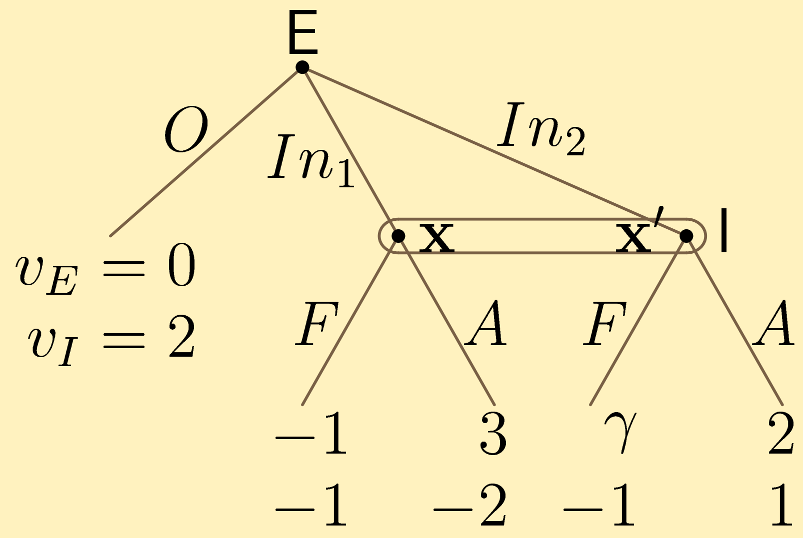
$$E[v_I(F,\sigma_E)|\mu] \gtreqless E[v_I(A,\sigma_E)|\mu] \Leftrightarrow \mu(\mathbf{x}) \gtreqless 2/3.$$

$*$ If $\gamma < 0$, $\sigma_E(O) = 1$ and $\sigma_I(F) = 1$ with $\mu(\mathbf{x}) \geq 2/3$.

$\mu(\mathbf{x}) < 2/3 \Rightarrow \sigma_I(F) = 0 \Rightarrow \sigma_E(In_1) = 1 \Rightarrow \mu(\mathbf{x}) = 1.$

Example

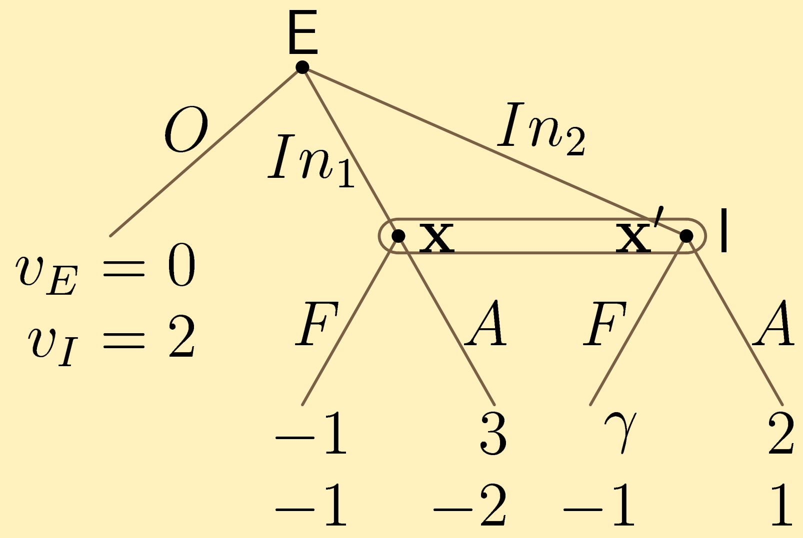
Example 9.C.3 $(\gamma > -1, \gamma \neq 0)$



$*$ If $\gamma < 0$, $\sigma_E(O) = 1$ and $\sigma_I(F) = 1$ with $\mu(\mathbf{x}) \geq 2/3$.

$\mu(\mathbf{x}) = 2/3$, then $E[v_E(\sigma_I, In_1)|\mu] = E[v_E(\sigma_I, In_2)|\mu]$.

Example 9.C.3 $(\gamma > -1, \gamma \neq 0)$



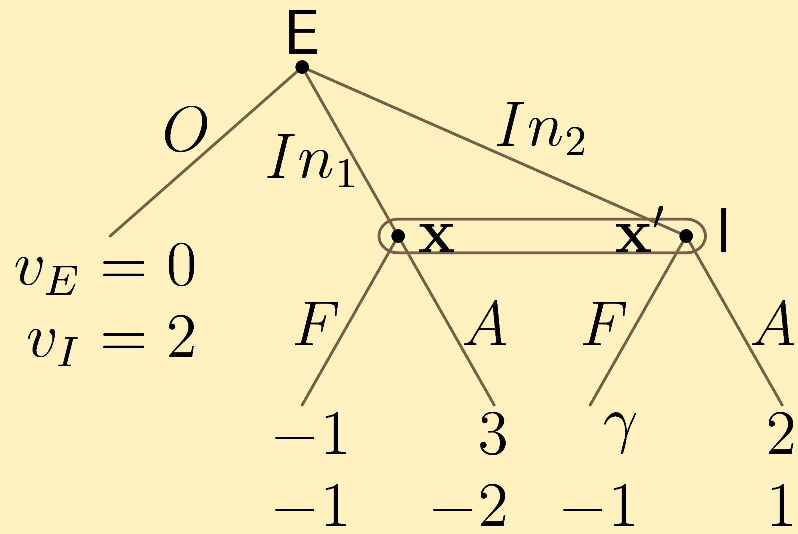
$*$ If $\gamma < 0$, $\sigma_E(O) = 1$ and $\sigma_I(F) = 1$ with $\mu(\mathbf{x}) \geq 2/3$.

$\mu(\mathbf{x}) = 2/3$, then $E[v_E(\sigma_I, In_1)|\mu] = E[v_E(\sigma_I, In_2)|\mu]$.

$\sigma_I(F) \times (-1) + (1 - \sigma_I(F)) \times 3 = \sigma_I(F) \times \gamma + (1 - \sigma_I(F)) \times 2.$

Example

Example 9.C.3 $(\gamma > -1, \gamma \neq 0)$



$*$ If $\gamma < 0$, $\sigma_E(O) = 1$ and $\sigma_I(F) = 1$ with $\mu(\mathbf{x}) \geq 2/3$.

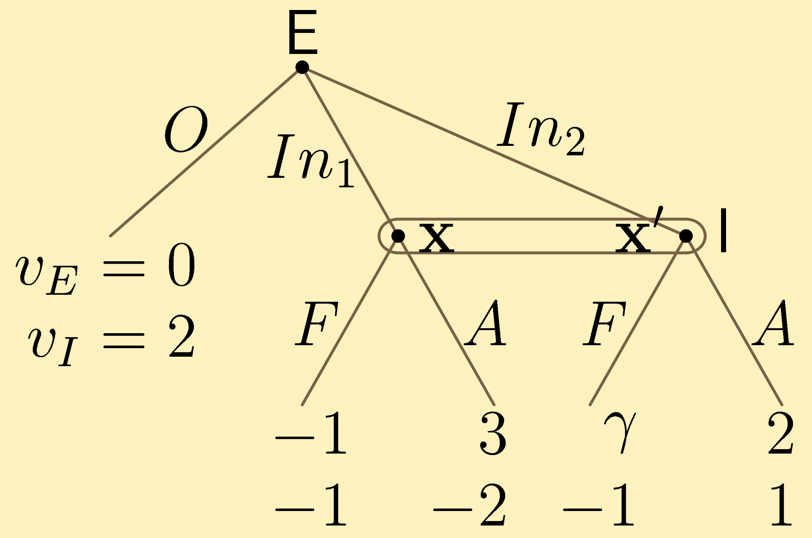
$\sigma_I(F) \times (-1) + (1 - \sigma_I(F)) \times 3 = \sigma_I(F) \times \gamma + (1 - \sigma_I(F)) \times 2.$

Solving it wrt $\sigma_I(F)$, we obtain

$\sigma_I(F) = 1/(\gamma + 2),\ \ E[v_E(\sigma_I, In_1)|\mu] = (3\gamma + 2)/(\gamma + 2).$

Example

Example 9.C.3 $(\gamma > -1,\ \gamma \neq 0)$



$*$ If $\gamma < 0$, $\sigma_E(O) = 1$ and $\sigma_I(F) = 1$ with $\mu(\mathbf{x}) \geq 2/3$.

$$\sigma_I(F) \times (-1) + (1 - \sigma_I(F)) \times 3 = \sigma_I(F) \times \gamma + (1 - \sigma_I(F)) \times 2.$$

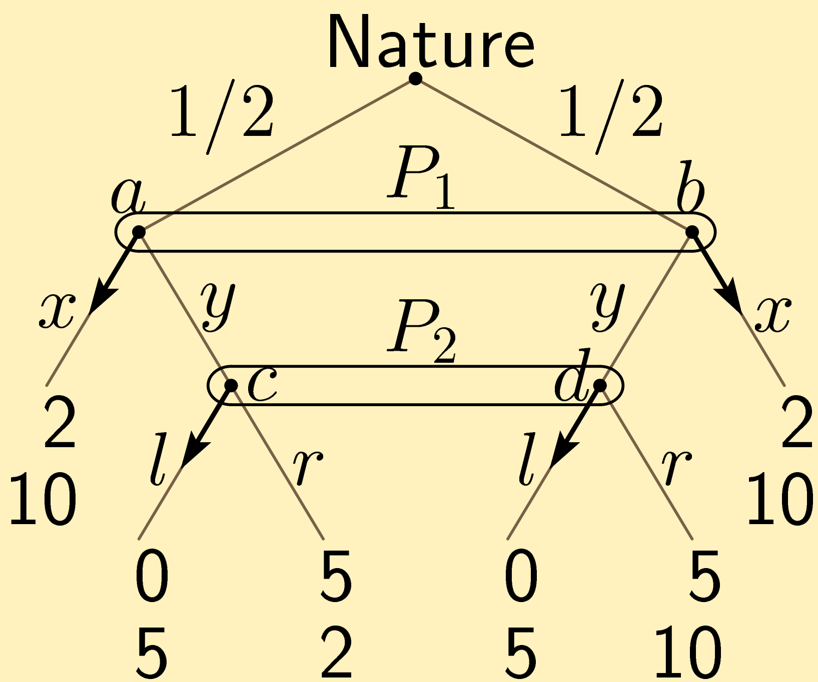
$$\sigma_I(F) = 1/(\gamma + 2),\ \ E[v_E(\sigma_I, In_1)|\mu] = (3\gamma + 2)/(\gamma + 2).$$

$*$ If $\gamma \geq -2/3$, $\sigma_E(O) = 0$, $\sigma_E(In_1) = 2/3$, $\sigma_E(In_2) = 1/3$.

Strengthening the wPBE concept (9.C.4)

$$\mu(a) = 1/2,\ \mu(b) = 1/2,\ \mu(c) = 9/10,\ \mu(d) = 1/10.$$

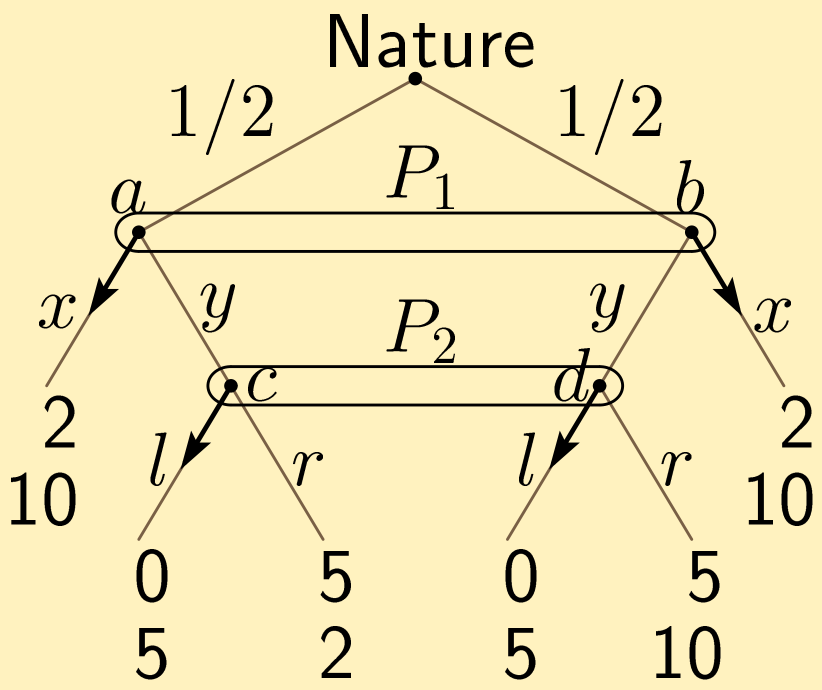
The arrows in the figure indicate the strategies.



Strengthening the wPBE concept (9.C.4)

$$\mu(a) = 1/2,\ \mu(b) = 1/2,\ \mu(c) = 9/10,\ \mu(d) = 1/10.$$

The arrows in the figure indicate the strategies.



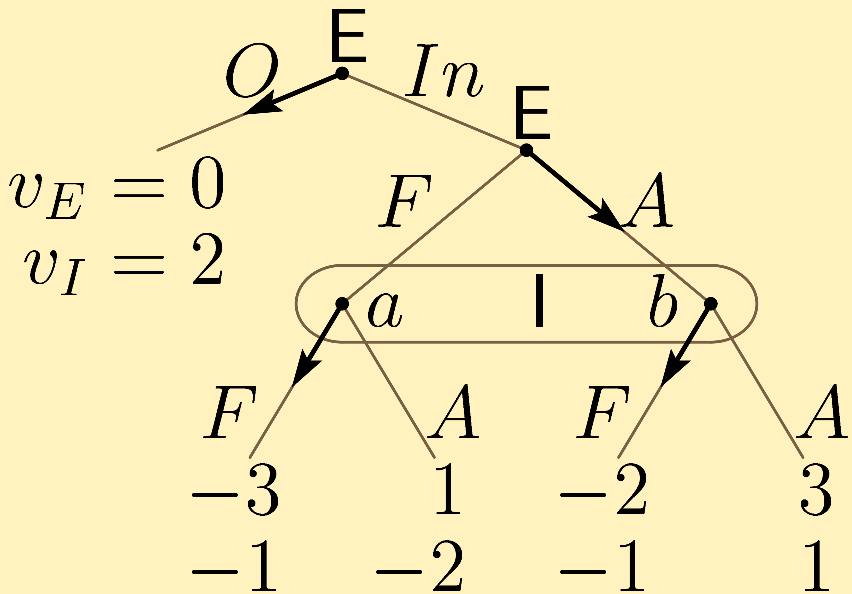
No restrictions at all are placed on beliefs **off the equilibrium path** (see Req. 15-3).

However, $\mu(c) = 9/10$ seems to be structurally inconsistent.

Strengthening the wPBE concept (9.C.5)

$$\mu(a) = 1,\ \mu(b) = 0.$$

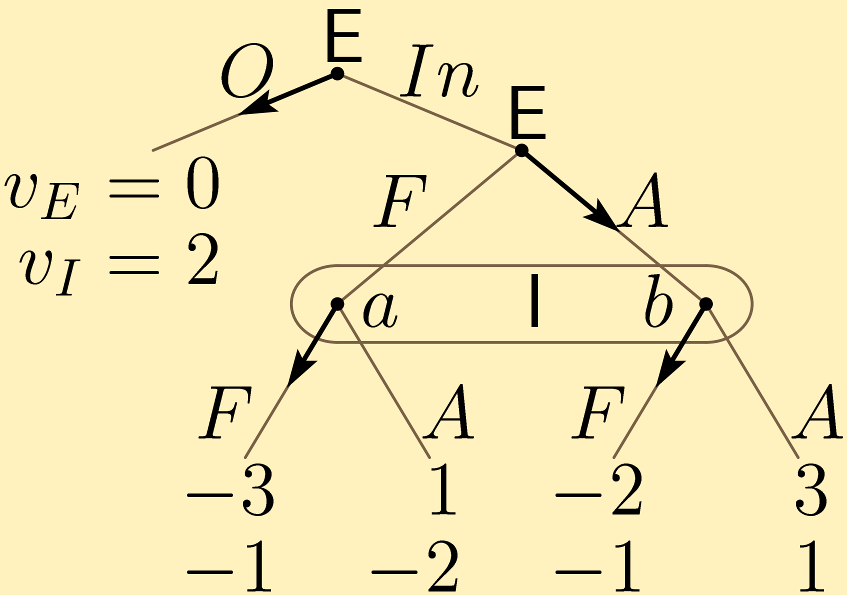
The arrows in the figure indicate the strategies.



Strengthening the wPBE concept (9.C.5)

$$\mu(a) = 1, \; \mu(b) = 0.$$

The arrows in the figure indicate the strategies.



E/I	F	A
F	$-3, \mathbf{-1}$	$1, -2$
A	$\mathbf{-2}, -1$	$\mathbf{3}, \mathbf{1}$

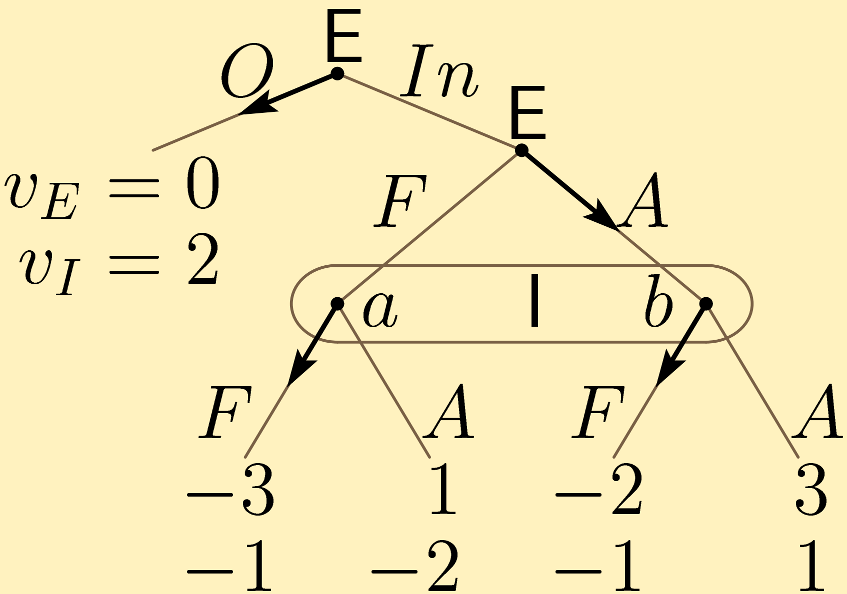
No restrictions at all are placed on beliefs **off the equilibrium path**.

Strengthening the wPBE concept

Strengthening the wPBE concept (9.C.5)

$$\mu(a) = 1,\ \mu(b) = 0.$$

The arrows in the figure indicate the strategies.



E/I	F	A
F	$-3, -\mathbf{1}$	$1, -2$
A	$-\mathbf{2}, -1$	$\mathbf{3}, \mathbf{1}$

No restrictions at all are placed on beliefs **off the equilibrium path**.

This outcome is NOT a SPNE outcome.

Sequential equilibrium

Definition 15.4 A system of beliefs μ is consistent with σ if there is a sequence $\{\sigma^k\}_{k=1}^{\infty}$ of total mixed strategies such that

1. $\lim_{k\to\infty} \sigma^k = \sigma$, 2. $\mu = \lim_{k\to\infty} \mu^k$,

where for all k, μ^k is the system of beliefs derived from σ^k by Bayes' rule.

Sequential equilibrium

Definition 15.4 A system of beliefs μ is consistent with σ if there is a sequence $\{\sigma^k\}_{k=1}^{\infty}$ of total mixed strategies such that

1. $\lim_{k\to\infty} \sigma^k = \sigma$, 2. $\mu = \lim_{k\to\infty} \mu^k$,

where for all k, μ^k is the system of beliefs derived from σ^k by Bayes' rule.

Definition 15.5 A pair (σ, μ) of strategy profile and system of beliefs is a sequential equilibrium if

1. σ is sequential rational given μ.
2. μ is consistent with σ.

Sequential equilibrium

Definition 15.4 A system of beliefs μ is consistent with σ if there is a sequence $\{\sigma^k\}_{k=1}^{\infty}$ of total mixed strategies such that

1. $\lim_{k\to\infty} \sigma^k = \sigma$, 2. $\mu = \lim_{k\to\infty} \mu^k$,

Definition 15.5 A pair (σ, μ) of strategy profile and system of beliefs is a sequential equilibrium if

1. σ is sequential rational given μ.
2. μ is consistent with σ.

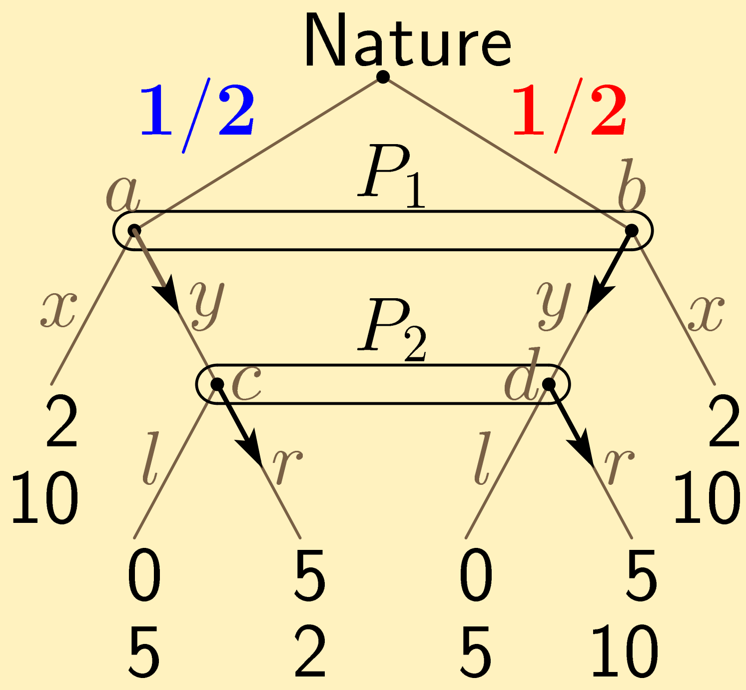
Prop. 9.C.2 In every sequential equilibrium (σ, μ) of Γ_E, a strategy profile σ is a subgame perfect Nash equilibrium of Γ_E

Example

Ex. 9.C.4 Reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

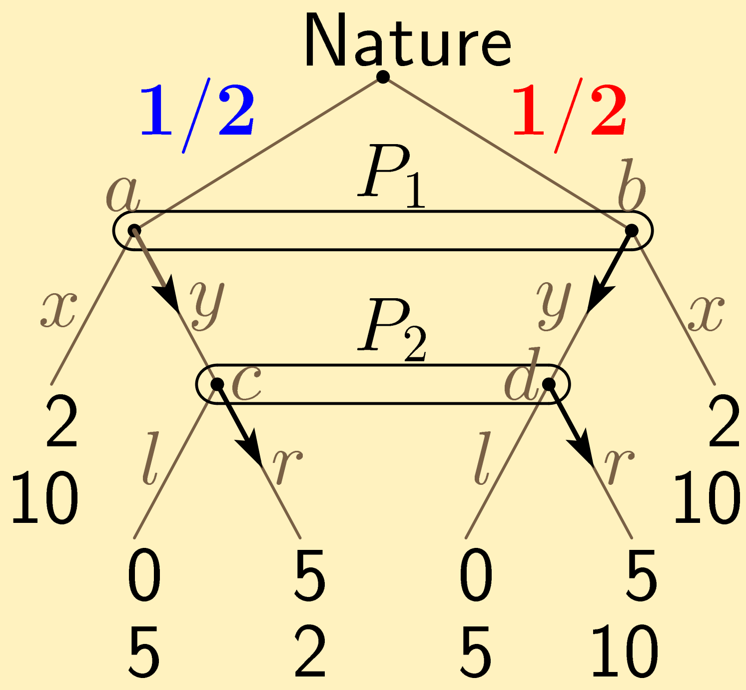
$$\forall k,\ \mu^k(a) = \frac{1}{2} \text{ and } \mu^k(c) = \frac{0.5\sigma_1^k(y)}{0.5\sigma_1^k(y) + 0.5\sigma_1^k(y)} = \frac{1}{2}.$$



Ex. 9.C.4 Reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{1}{2} \text{ and } \mu^k(c) = \frac{0.5\sigma_1^k(y)}{0.5\sigma_1^k(y) + 0.5\sigma_1^k(y)} = \frac{1}{2}.$$

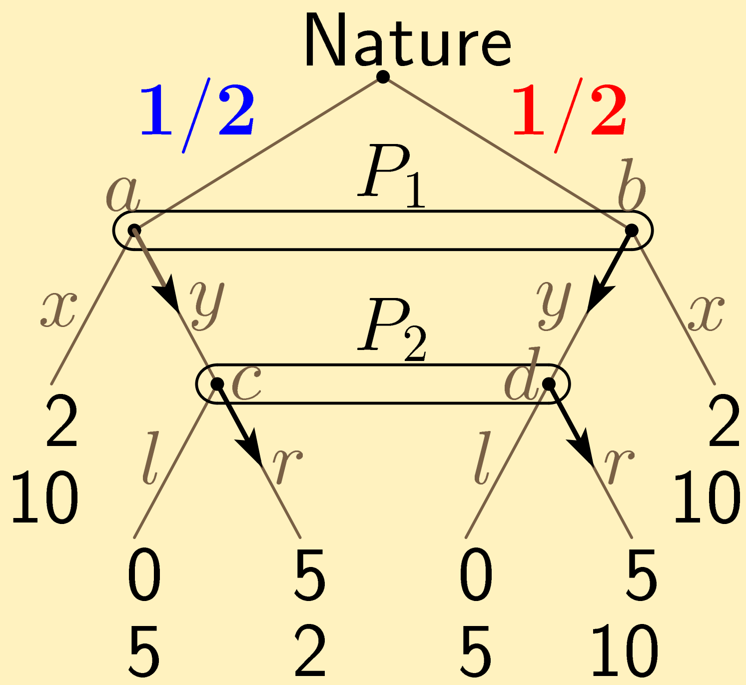


$\lim_{k\to\infty} \mu^k(a) = 1/2,$
$\lim_{k\to\infty} \mu^k(c) = 1/2.$

Ex. 9.C.4 Reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{1}{2} \text{ and } \mu^k(c) = \frac{0.5\sigma_1^k(y)}{0.5\sigma_1^k(y) + 0.5\sigma_1^k(y)} = \frac{1}{2}.$$



$\lim_{k\to\infty} \mu^k(a) = 1/2,$
$\lim_{k\to\infty} \mu^k(c) = 1/2.$

$E(u_2|H_2, \mu, \mathbf{r}, \sigma_1) > E(u_2|H_2, \mu, \mathbf{l}, \sigma_1).$

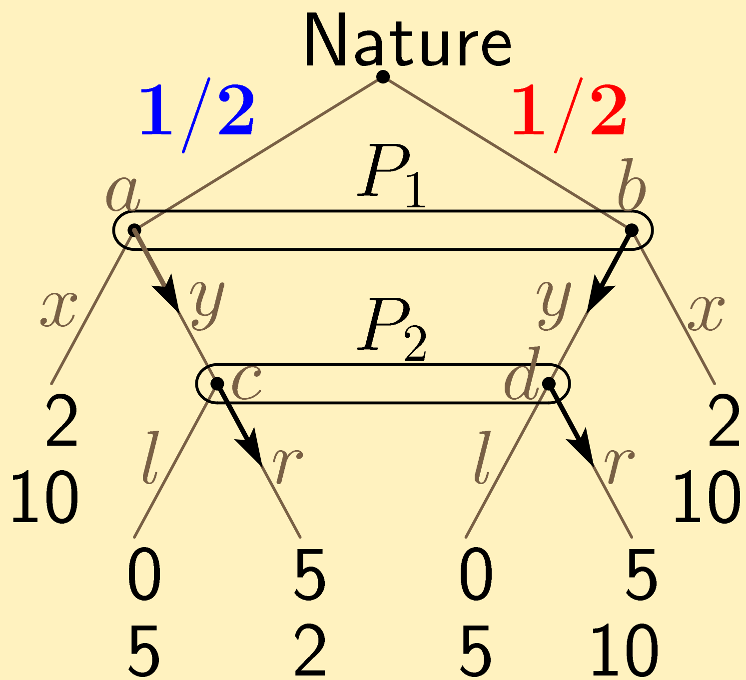
$\Rightarrow \sigma_2(r) = 1,\ \sigma_2(l) = 0.$

Example

Ex. 9.C.4 Reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{1}{2} \text{ and } \mu^k(c) = \frac{0.5\sigma_1^k(y)}{0.5\sigma_1^k(y) + 0.5\sigma_1^k(y)} = \frac{1}{2}.$$



$\lim_{k\to\infty} \mu^k(a) = 1/2,$
$\lim_{k\to\infty} \mu^k(c) = 1/2.$

$E(u_2|H_2, \mu, \mathbf{r}, \sigma_1) > E(u_2|H_2, \mu, \mathbf{l}, \sigma_1).$
$\Rightarrow \sigma_2(r) = 1,\ \sigma_2(l) = 0.$

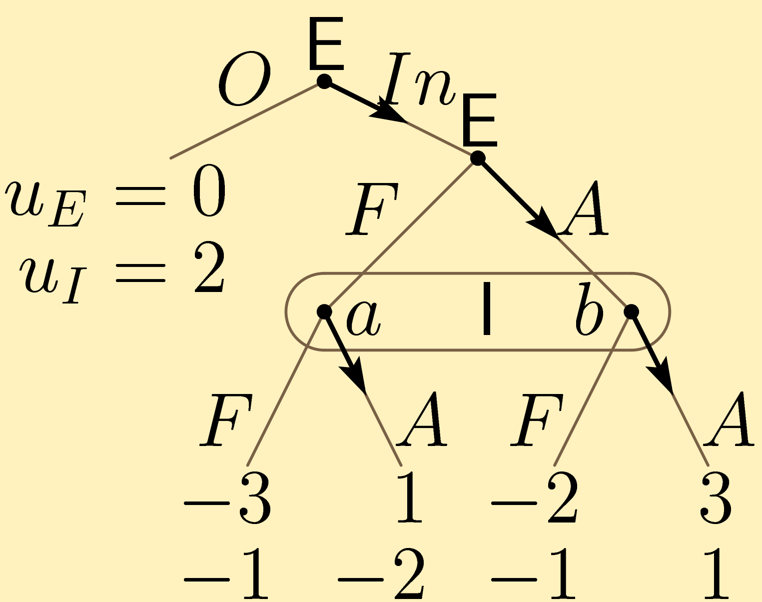
$E(u_1|H_1, \mu, \mathbf{y}, \sigma_2) > E(u_1|H_1, \mu, \mathbf{x}, \sigma_2).$
$\Rightarrow \sigma_1(y) = 1,\ \sigma_1(x) = 0.$

Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$

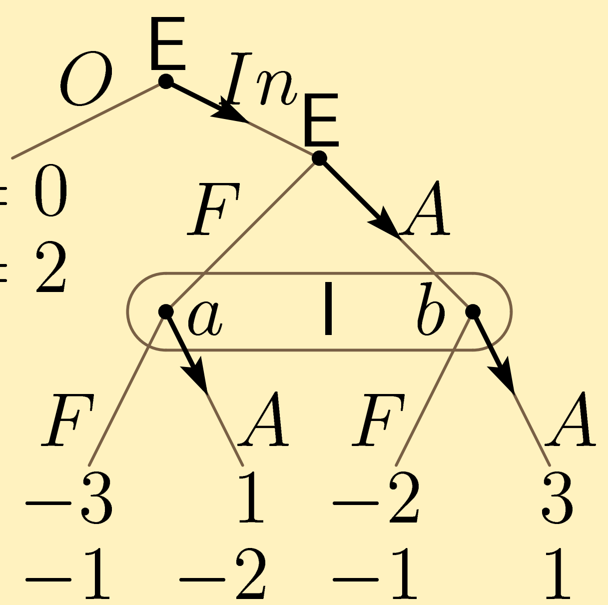


Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$



$u_E = 0$
$u_I = 2$

$$E(u_E|H_2, \mu, \sigma_E(In), \mathbf{F}, \sigma_I) < E(u_E|H_2, \mu, \sigma_E(In), \mathbf{A}, \sigma_I).$$

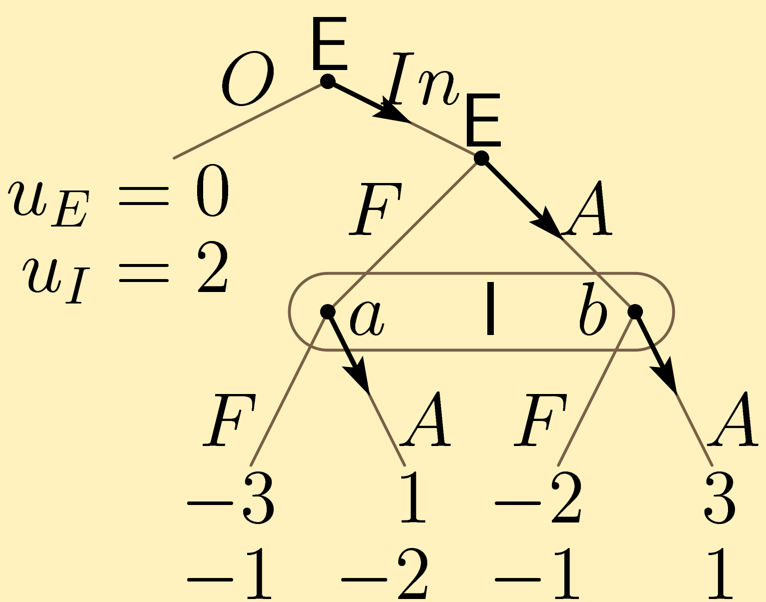
Example

Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$



$$E(u_E|H_2, \mu, \sigma_E(In), \mathbf{F}, \sigma_I) < E(u_E|H_2, \mu, \sigma_E(In), \mathbf{A}, \sigma_I).$$

$\Rightarrow \sigma_E(F) = 0,\ \sigma_E(A) = 1.$

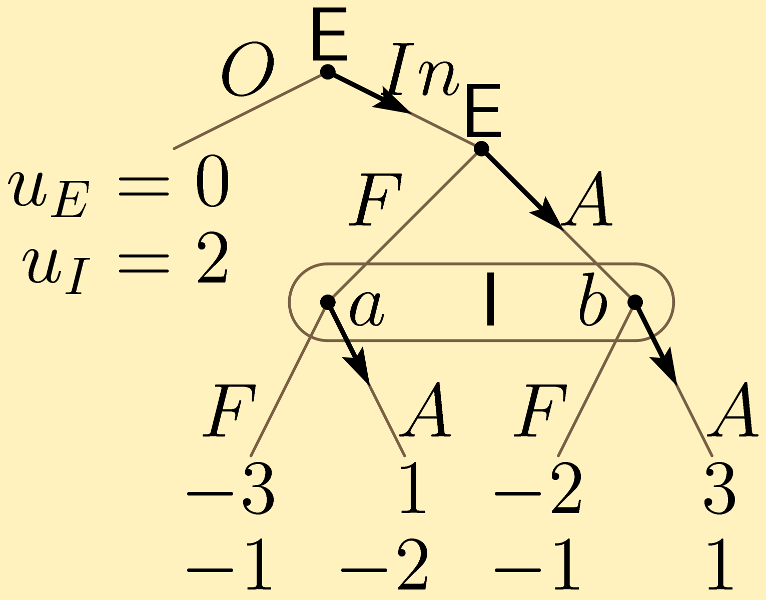
Example

Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$



$\sigma_E(F) = 0$, $\sigma_E(A) = 1$.

$E(u_I|H_I, \mu, \sigma_E, \mathbf{F}) < E(u_I|H_I, \mu, \sigma_E, \mathbf{A})$.

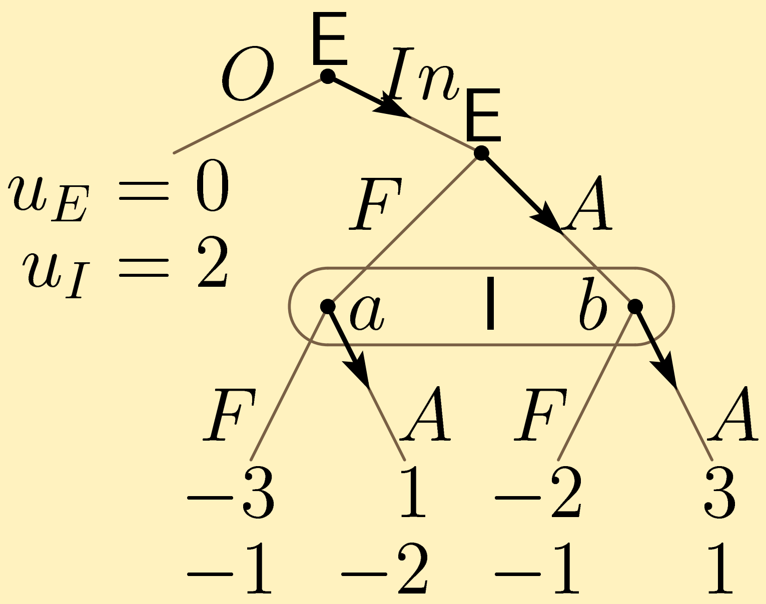
Example

Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$



$\sigma_E(F) = 0$, $\sigma_E(A) = 1$.

$E(u_I|H_I, \mu, \sigma_E, \mathbf{F}) < E(u_I|H_I, \mu, \sigma_E, \mathbf{A})$.

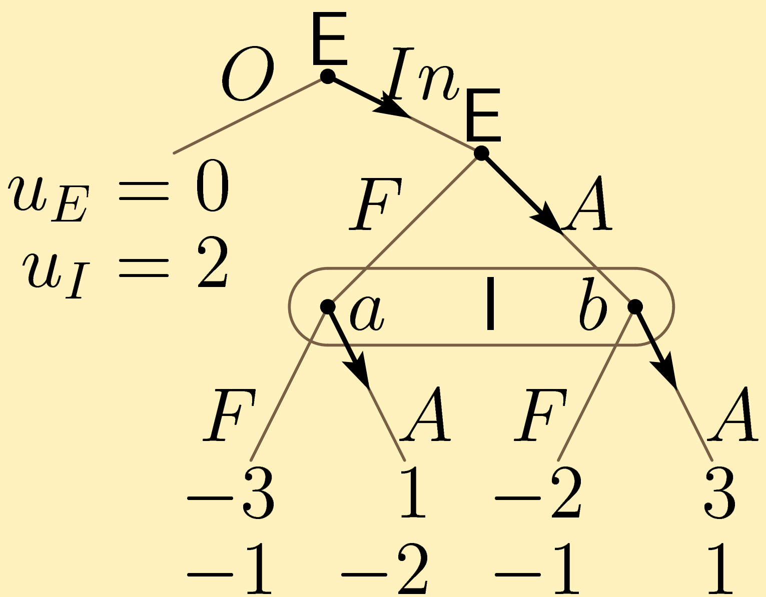
$\Rightarrow \sigma_I(F) = 0$, $\sigma_I(A) = 1$.

Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$



$\sigma_E(F) = 0$, $\sigma_E(A) = 1$.

$\sigma_I(F) = 0$, $\sigma_I(A) = 1$.

$E(u_E|H_1, \mu, \sigma_I, \mathbf{Out})$

$< E(u_E|H_1, \mu, \sigma_I, \mathbf{In})$.

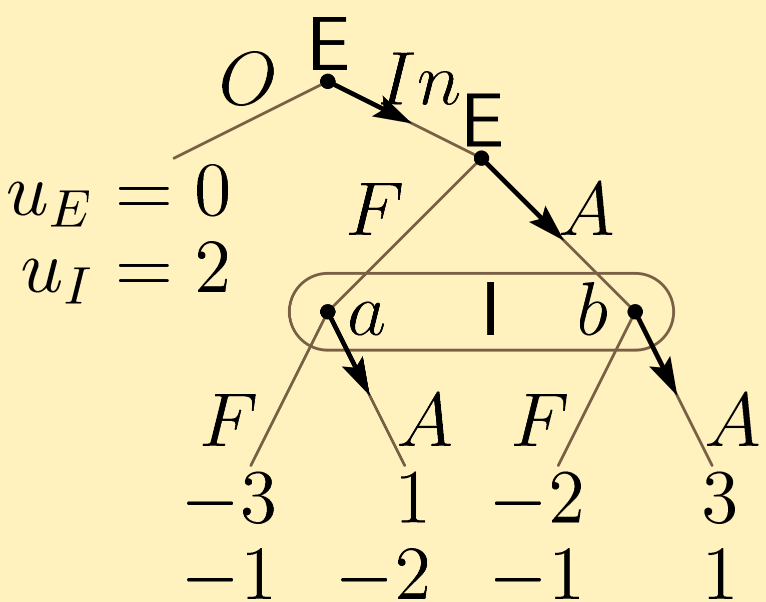
Example

Ex. 9.C.5 reconsidered

Let (σ, μ) be a sequential equilibrium. Let $\sigma^k \to \sigma$.

$$\forall k,\ \mu^k(a) = \frac{\Pr(a|\sigma^k)}{\Pr(H_I|\sigma^k)} = \frac{\sigma_E^k(In) \times \sigma_E^k(F)}{\sigma_E^k(In)} = \sigma_E^k(F).$$

$$\mu(a) = \lim_{k\to\infty} \mu^k(a) = \lim_{k\to\infty} \sigma_E^k(F) = \sigma_E(F).$$



$\sigma_E(F) = 0,\ \sigma_E(A) = 1.$

$\sigma_I(F) = 0,\ \sigma_I(A) = 1.$

$E(u_E|H_1, \mu, \sigma_I, \mathbf{Out})$

$< E(u_E|H_1, \mu, \sigma_I, \mathbf{In}).$

$((\sigma_E, \sigma_I), (\mu(a), \mu(b)))$

$= (((In, A), A), (0, 1)).$