Chapter 13 (§13C,D): Signaling, and Screening in Mas-Colell et al.
Chapter 16: Limit pricing in Tadelis

- Signaling (§13.C in MWG)
- Screening (§13.D in MWG)
- Limit pricing (§16.2 in Tadelis)
  A slightly generalized version based on Belleflamme and Peitz (2010)

§13.C Signaling

Basic assumptions

1. High type workers obtain degrees with low costs.
2. Low type workers obtain degrees with high costs.
3. Education has no effect on workers' productivity.

$\Theta = \{\theta_H, \theta_L\}$, $\theta_H > \theta_L > 0$, $\lambda = \Pr(\theta = \theta_H) \in (0, 1)$.

For simplicity, there are only two types of workers.

§13.C Signaling

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$\Theta = \{\theta_H, \theta_L\}$, $\theta_H > \theta_L > 0$, $\lambda = \Pr(\theta = \theta_H) \in (0, 1)$.
$e \in [0, \infty)$: education level,
$C(e, \theta)$: type θ's cost to obtain e.

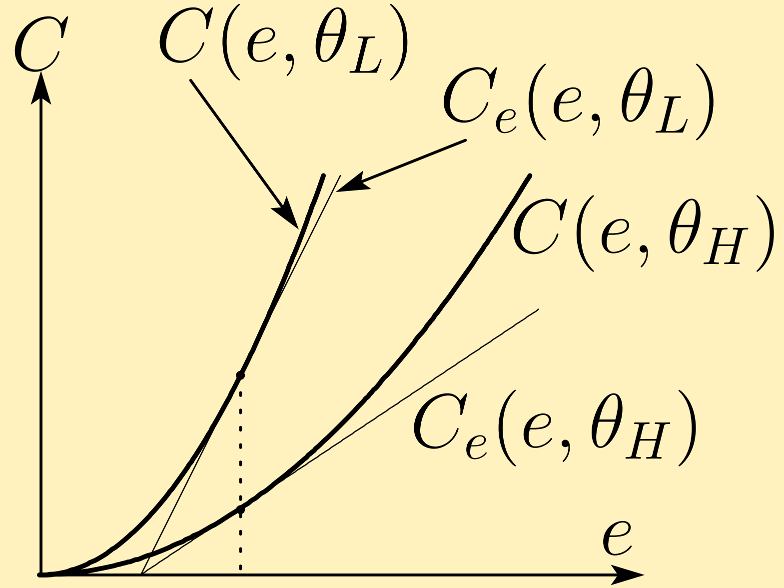
Basic assumptions (cont.)

$\forall \theta$, $C(0, \theta) = 0$.

$\forall \theta$, $\forall e$, $C_e(e, \theta) > 0$.

$\forall \theta$, $\forall e$, $C_{ee}(e, \theta) > 0$.

$\forall e$, $C(e, \theta_H) < C(e, \theta_L)$.



Single-crossing property: $\forall e$, $C_e(e, \theta_H) < C_e(e, \theta_L)$.

Signaling game

Basic assumptions (cont.)

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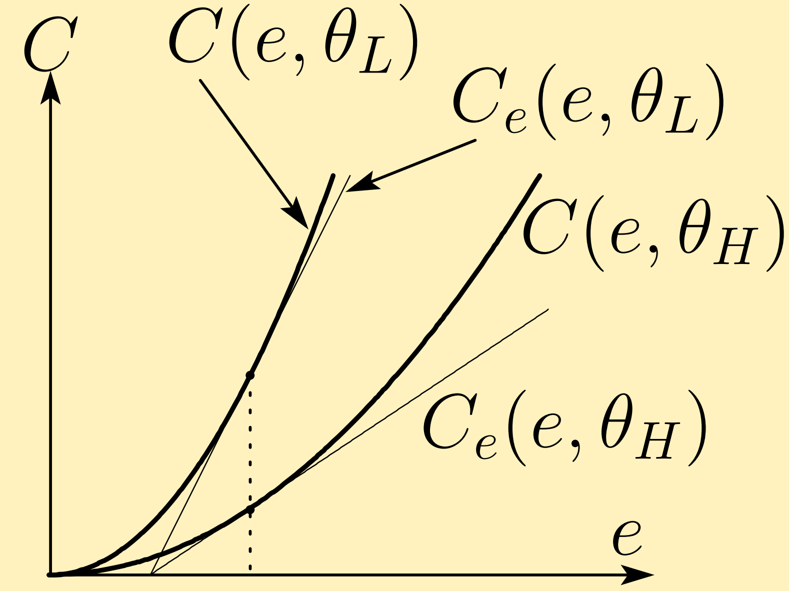


Single-crossing property: $\forall e$, $C_e(e,\theta_H) < C_e(e,\theta_L)$.

Reservation payoff: $r(\theta_H) = r(\theta_L) = 0$.

Workers' payoff: $v(w, e|\theta) = w - C(e,\theta)$.

$\mu(e)$: The firm's belief that a worker is of type θ_H after it observes e.

Signaling game

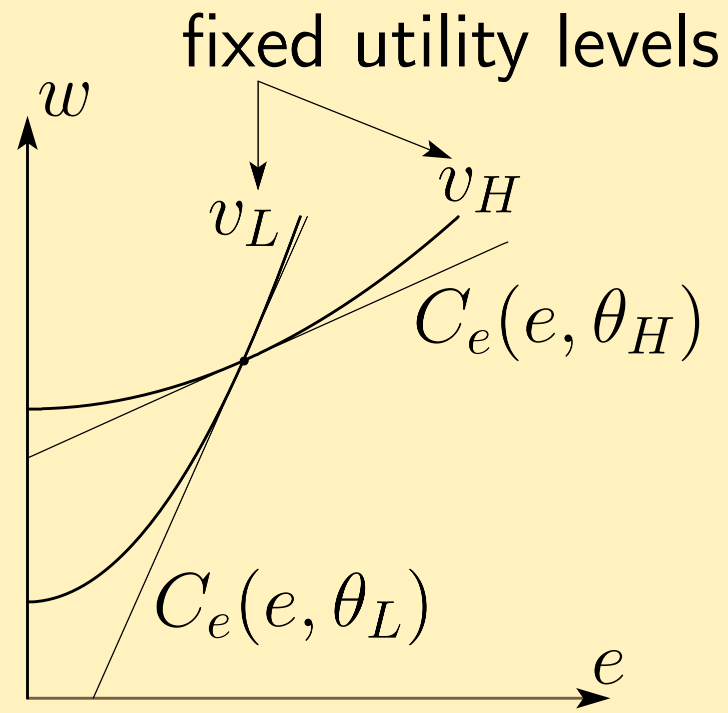
Single-crossing property (SCP) The indifference curves of the two types cross at most once.

$\forall e$, $C_e(e, \theta_H) < C_e(e, \theta_L)$

Indifference curve: $w = \bar{v} + C(e, \theta)$.

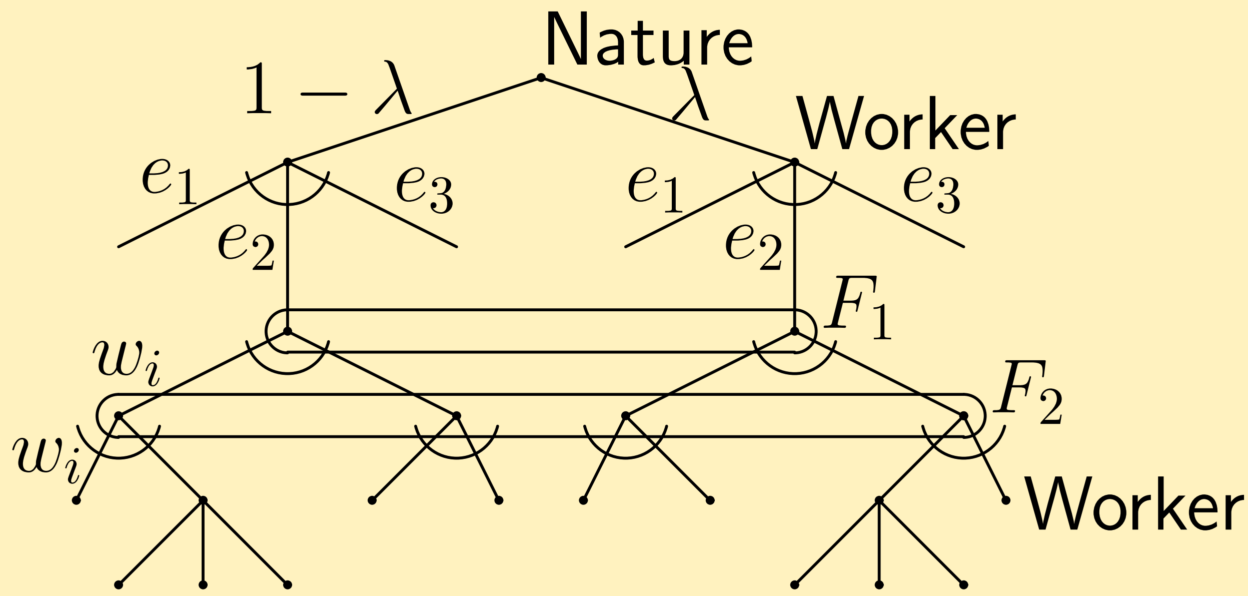
MRS: $\left.\frac{dw}{de}\right|_{v:const.} = C_e(e, \theta)$.

$\partial \text{MRS}/\partial\theta = C_{e\theta}(e, \theta) < 0$.



Signaling game

The timing structure of the game



1. Nature determines the worker's ability, θ_H or θ_L.
2. Observing the ability, the worker determines e.
3. Observing e, each firm simultaneously offers w_i.
4. Observing the wages, the worker decides whether to work for a firm, if so, which one.

Signaling game

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In the next slide, we check the basic equilibrium property.

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   (1) The worker's strategy is optimal given the firms' strategies.

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Remark Let e_k be type θ_k's choice $(k = H, L)$ in PBE.

1. In the final stage, a worker will accept $\max\{w_1(e), w_2(e)\}$ given his choice e.
2. $\mu(e_H) = \lambda/\lambda = 1$ if $e_H \neq e_L$, and $\mu(e_H) = \lambda$ (*ex ante* prob. he/she is H) if $e_H = e_L$.

   (2) $\mu(e)$ is derived from the worker's strategy using Bayes' rule where possible.

   Observing the realized e, firms can find the worker's type if $e_H \neq e_L$.

Signaling game

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2. $\mu(e_H) = \lambda/\lambda = 1$ if $e_H \neq e_L$, and $\mu(e_H) = \lambda$ (*ex ante* prob. he/she is H) if $e_H = e_L$.
3. Given e, the firms' wage offers are those of the standard Bertrand model, so that $w_1(e) = w_2(e) = E(\theta; e) = \mu(e)\theta_H + (1 - \mu(e))\theta_L$.

   (3) The firms' wage offers $(w_1(e), w_2(e))$ following each e constitute a Nash equilibrium of the simultaneous move wage offer game in which the probability that the worker is of θ_H is $\mu(e)$.

Equilibria

What we do here Check the equilibrium properties of the following two.

Two types of equilibria

1. Separating: $e^*(\theta_H) \neq e^*(\theta_L)$.
2. Pooling: $e^*(\theta_H) = e^*(\theta_L)$.

$e^*(\theta)$ denotes an education choice function in a PBE.
$w^*(e)$ denotes a wage offer function in a PBE.

Separating equilibrium

Separating equilibria

Lemma 13.C.1 In any separating PBE,

$$w^*(e^*(\theta_H)) = \theta_H \text{ and } w^*(e^*(\theta_L)) = \theta_L.$$

If $e^*(\theta_H) \neq e^*(\theta_L)$, $\mu(e^*(\theta_H)) = 1$ and $\mu(e^*(\theta_L)) = 0$.

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Then,
$w^*(e^*(\theta_H)) = E(\theta|e^*(\theta_H)) = \theta_H$ and
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Lemma 13.C.2 In any separating PBE, $e^*(\theta_L) = 0$.

The utility level of type L is $\theta_L - 0 = \theta_L$.

Separating equilibrium

Separating equilibria

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Then,
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$w^*(e^*(\theta_L)) = E(\theta|e^*(\theta_L)) = \theta_L$.

Lemma 13.C.2 In any separating PBE, $e^*(\theta_L) = 0$.

Proof: By contradiction. Suppose that $e^*(\theta_L) > 0$.

From Lemma 13.C.1, type θ_L gains $\theta_L - C(e^*(\theta_L), \theta_L)$.

Choosing $e^*(\theta_L) = 0$ improves his/her gain.

Separating equilibrium

Lemma 13.C.2 Type L with $e(\theta_L) = 0$ accepts θ_L.

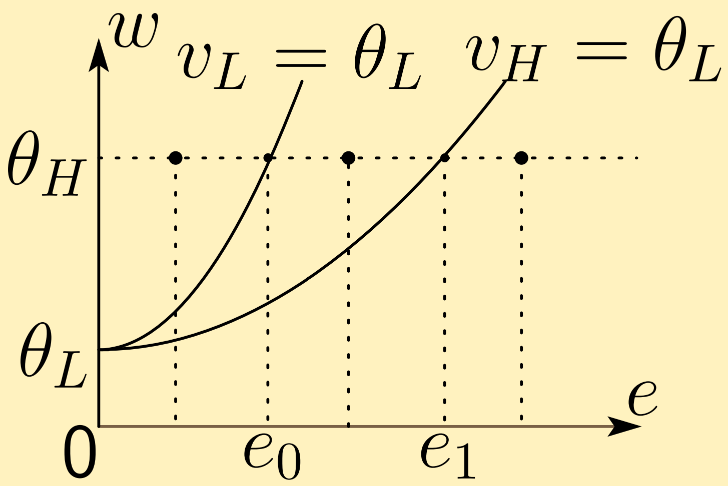
Effort levels Let e_0 and e_1 be such that

$$v_L = \theta_L = \theta_H - C(e_0, \theta_L),$$
$$v_H = \theta_L = \theta_H - C(e_1, \theta_H).$$

(See $(e, w) = (0, \theta_L)$).

Assume that

$e_H^* = e^*(\theta_H)$, $e_L^* = e^*(\theta_L)$ in PBE.



Fact 13.C.1 In any separating PBE, $e_0 \le e_H^* \le e_1$.

Type H: $(\theta_H, e_H^*) \succ (\theta_L, 0)$

Type L: $(\theta_H, e_H^*) \prec (\theta_L, 0)$

Separating equilibrium

Effort levels Let e_0 and e_1 be such that

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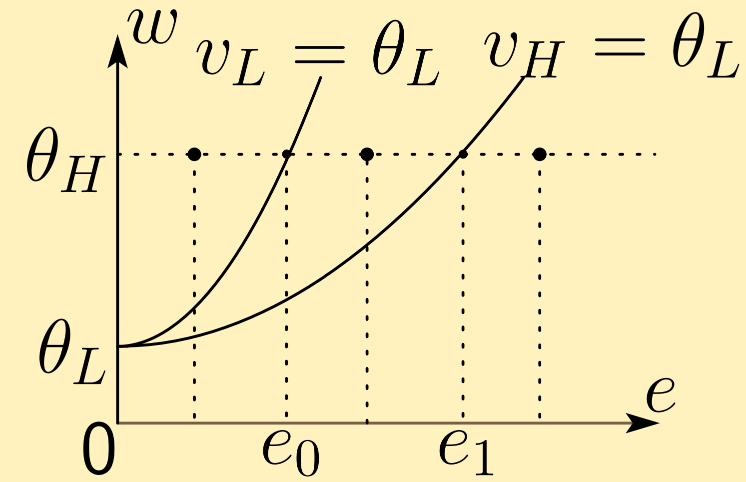

Fact 13.C.1 In any separating PBE, $e_0 \leq e_H^* \leq e_1$.

By Lemmas 1 and 2, $w(e_L^*) = \theta_L$, $w(e_H^*) = \theta_H$, $e_L^* = 0$.

Separating equilibrium

Effort levels Let e_0 and e_1 be such that

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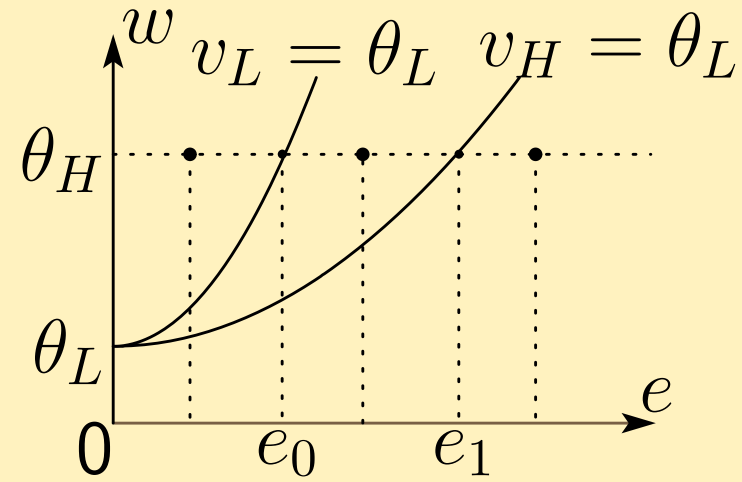

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By Lemmas 1 and 2, $w(e_L^*) = \theta_L$, $w(e_H^*) = \theta_H$, $e_L^* = 0$.

If $e_H^* < e_0$, $\theta_L < v_L = \theta_H - C(e_H^*, \theta_L)$. Type θ_L can be better off by choosing e_H^*.

Both types prefer $(w(e_H^*), e_H^*)$.

Separating equilibrium

Effort levels Let e_0 and e_1 be such that

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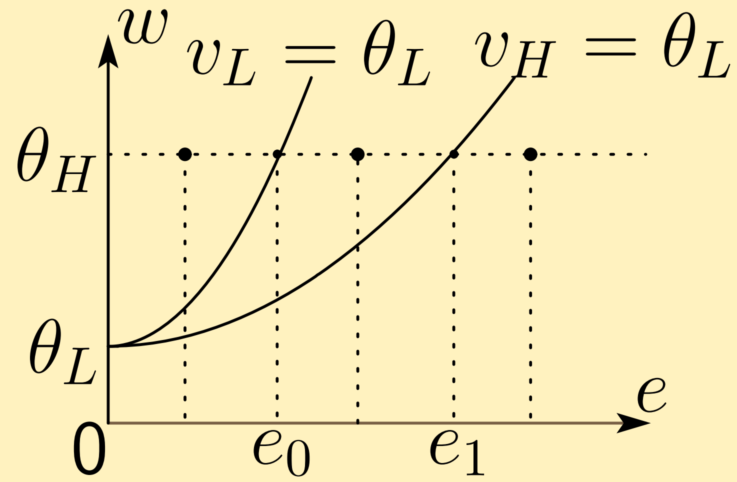

Fact 13.C.1 In any separating PBE, $e_0 \leq e_H^* \leq e_1$.

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If $e_H^* < e_0$, $\theta_L < v_L = \theta_H - C(e_H^*, \theta_L)$. Type θ_L can be better off by choosing e_H^*.

If $e_H^* > e_1$, $\theta_L > v_H = \theta_H - C(e_H^*, \theta_H)$. Type θ_H can be better off by choosing e_L^*. No type chooses e_H^*.

Separating equilibrium

Fact 13.C.2 In any PBE, μ^* is such that for all $\theta \in \Theta$,

$$\begin{aligned} e^*(\theta) &= \arg\max_e \{w^*(e) - C(e,\theta)\} \\ &= \arg\max_e \{[\mu^*(e)\theta_H + (1-\mu^*(e))\theta_L] - C(e,\theta)\}. \end{aligned}$$

Separating equilibrium

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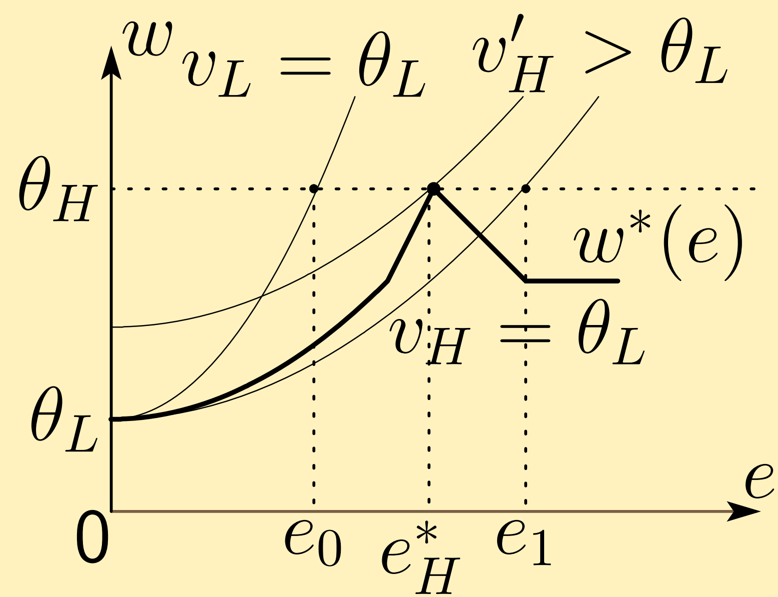
$$e^*(\theta) = \arg\max_e \{w^*(e) - C(e,\theta)\}$$
$$= \arg\max_e \{[\mu^*(e)\theta_H + (1-\mu^*(e))\theta_L] - C(e,\theta)\}.$$

Fact 13.C.3 The strategies and beliefs described in Remark, Lemmas 13.C.1 and 2, and Facts 13.C.1 and 2 constitute a separating PBE.

For e except $e_L^* = 0$ and $e = e_H^*$, we can arbitrary set $\mu^*(e)$ if the setting does not change the worker's choice.



Separating equilibrium

Ex. 1 $\theta_L = 1$, $\theta_H = 2$, $C(e, \theta_L) = e^2$, $C(e, \theta_H) = e^2/4$.

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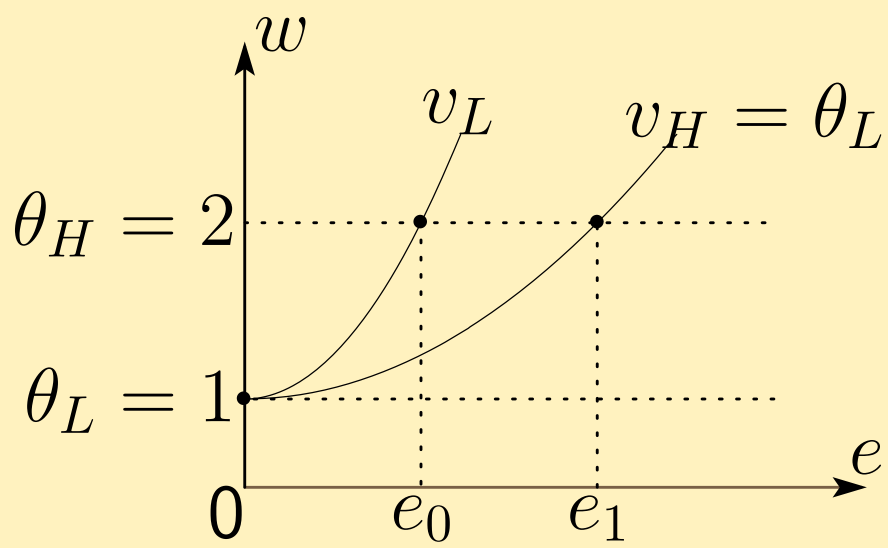
$$\theta_L - C(0, \theta_L) = \theta_H - C(e_0, \theta_L) \ \rightarrow \ e_0 = 1,$$
$$\theta_L - C(0, \theta_H) = \theta_H - C(e_1, \theta_H) \ \rightarrow \ e_1 = 2.$$



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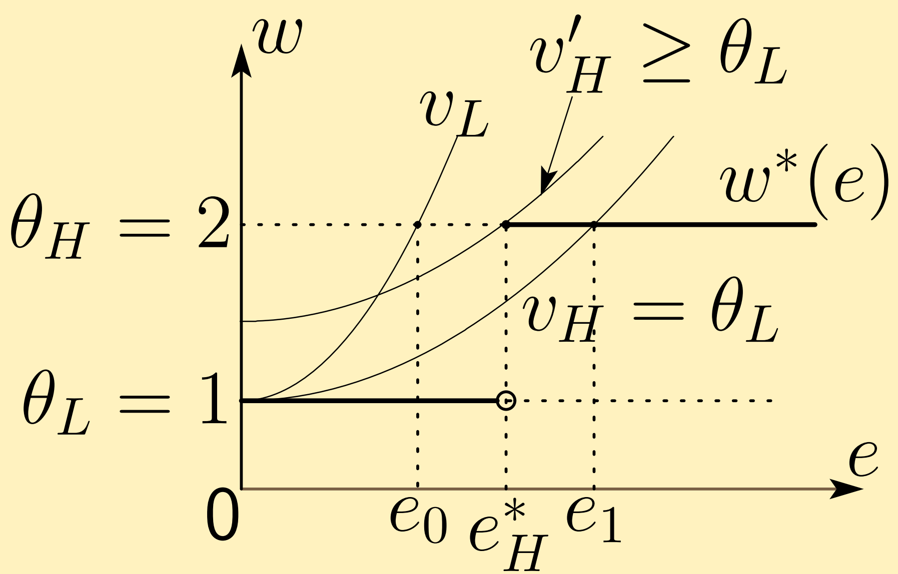
$$\theta_L - C(0, \theta_H) = \theta_H - C(e_1, \theta_H) \ \rightarrow \ e_1 = 2.$$

Let $e_H^* \in [1, 2]$, and let μ^* and $w^*(\cdot)$ be such that

$$\mu^*(e) = \begin{cases} 1 & \text{if } e \geq e_H^*, \\ 0 & \text{if } e < e_H^*. \end{cases}$$

$$w^*(e) = \begin{cases} \theta_H & \text{if } e \geq e_H^*, \\ \theta_L & \text{if } e < e_H^*. \end{cases}$$



Separating equilibrium

Fact 13.C.4 In any separating PBE, the profits of the firms are zero, type L worker obtains θ_L, type H worker obtains $\theta_H - C(e^*(\theta_H), \theta_H)$.

Separating equilibrium

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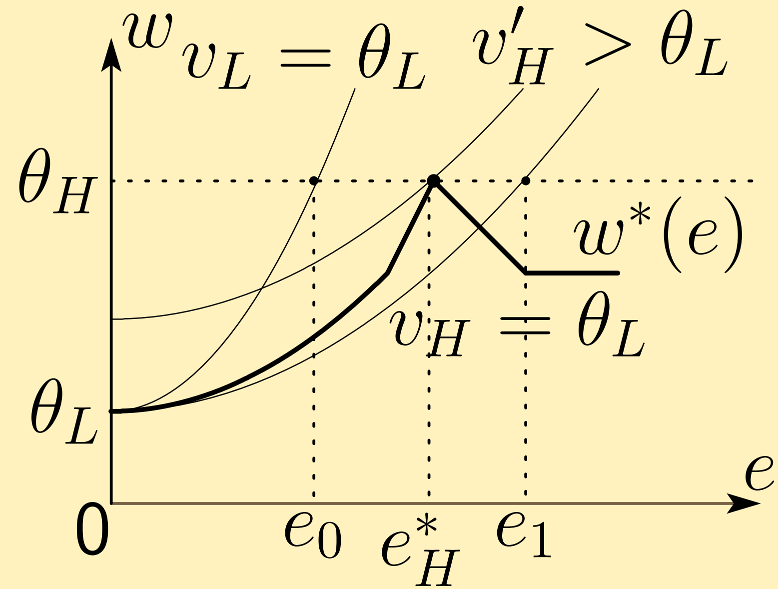
Fact 13.C.5 The separating PBE with $e^*(\theta_H) = e_0$ Pareto-dominates any other separating equilibria.

$e_0(+\varepsilon)$ is sufficient to distinguish the types.



Separating equilibrium

Fact 13.C.6 Consider a separating PBE with $e^*(\theta_H) = e^*_H$. Let $\bar{w} = \theta_H - C(e^*_H, \theta_H) = v_H$, and let α be such that $\alpha\theta_H + (1-\alpha)\theta_L = \bar{w}$.

If $\alpha < \lambda$, that is, if $\bar{w} < E(\theta) = \lambda\theta_H + (1-\lambda)\theta_L$, the allocation without signals Pareto-dominates the separating PBE.

Separating equilibrium

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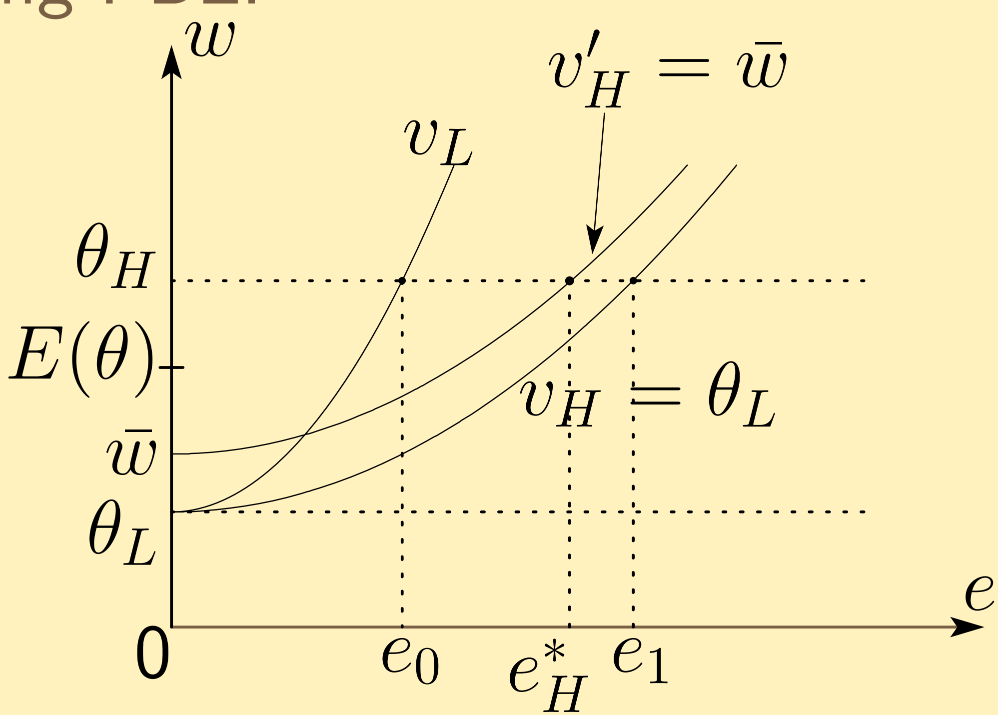
If $\alpha < \lambda$, the allocation without signals Pareto-dominates the separating PBE.



Pooling equilibrium

Pooling $e^* = e^*(\theta_L) = e^*(\theta_H)$ in a pooling PBE.

Pooling equilibrium

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Fact 13.C.7 In any pooling PBE,

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Pooling equilibrium

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Fact 13.C.7 In any pooling PBE,

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Fact 13.C.8 In any pooling PBE with $e^* = e^*(\theta_L)$,

the profits of the firms are zero,

type L worker obtains $E(\theta) - C(e^*, \theta_L)$,

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Fact 13.C.9 In any pooling PBE with $e^* = e^*(\theta_L)$,

$0 \leq e^* \leq e'$ where e' satisfies

$$E(\theta) - C(e^*, \theta_L) \geq E(\theta) - C(e', \theta_L) = \theta_L.$$

Pooling equilibrium

Pooling $e^* = e^*(\theta_L) = e^*(\theta_H)$ in a pooling PBE.

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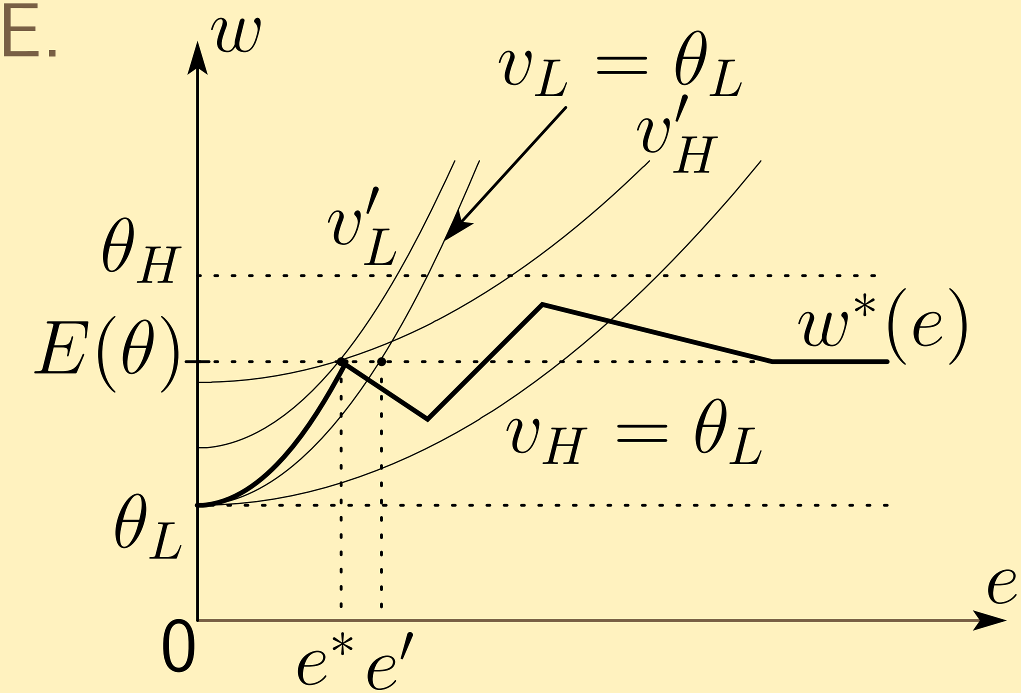
Fact 13.C.9 In any pooling PBE with $e^* = e^*(\theta_L)$,

$0 \le e^* \le e'$ where e' satisfies $E(\theta) - C(e', \theta_L) = \theta_L$.

Proof: If $e > e'$, then type L chooses $e = 0$ because he/she gains at least θ_L.

Pooling equilibrium

Fact 13.C.10 The strategies and beliefs described in Remark of def. of PBE, Facts 13.C.2, 7, and 9 constitute a pooling PBE.



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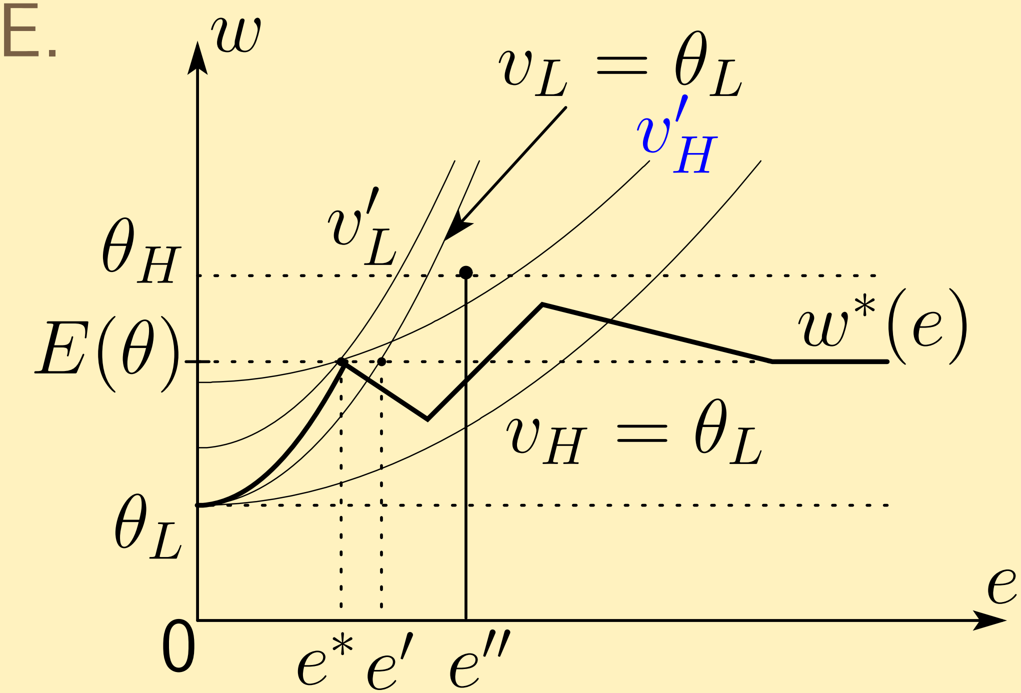

Given the firms' beliefs $\mu^*(e)$, no firm has an incentive to attract only type H, by offering θ_H for a worker with $e = e''$. $\mu^*(e)$ is not large for $e = e''$ ($w^*(e)$ reflects the belief).

Pooling equilibrium

Ex. 2 $\theta_L = 1$, $\theta_H = 2$, $C(e, \theta_L) = e^2$, $C(e, \theta_H) = e^2/4$, $\lambda = 1/2$.

Pooling equilibrium

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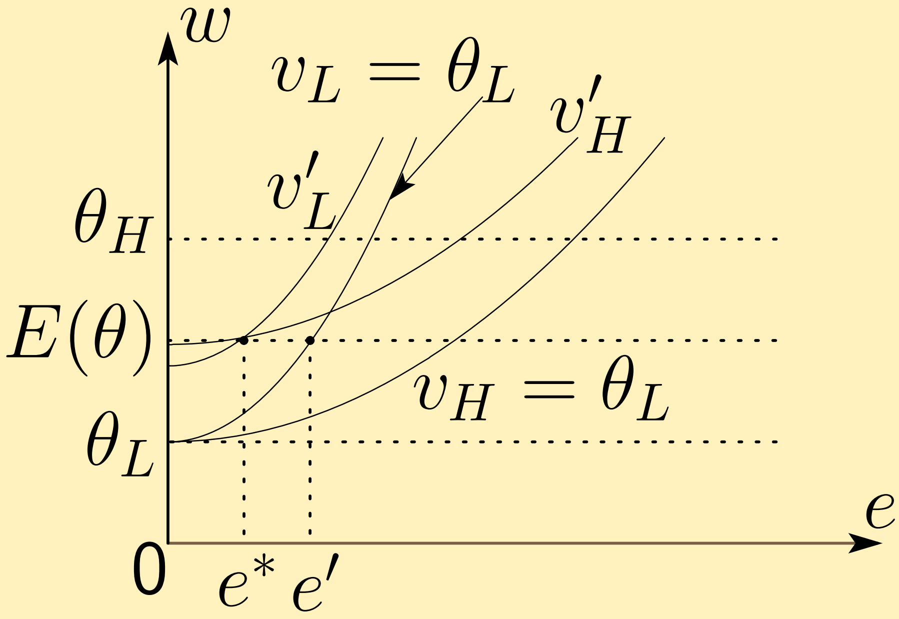
$E(\theta) = 1 \times (1/2) + 2 \times (1/2) = 3/2$.

$\theta_L = E(\theta) - C(e', \theta_L) \ \rightarrow \ e' = \sqrt{2}/2$.



Pooling equilibrium

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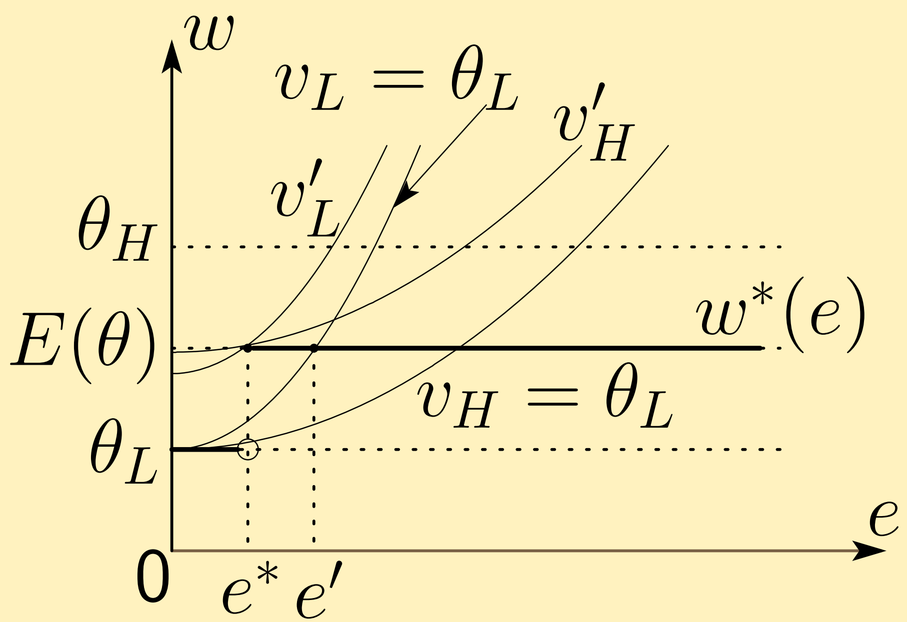
$E(\theta) = 1 \times (1/2) + 2 \times (1/2) = 3/2.$

$\theta_L = E(\theta) - C(e', \theta_L) \ \rightarrow \ e' = \sqrt{2}/2.$

Let $e^* \in [0, \sqrt{2}/2]$. Let μ^* and w^* be such that

$$\mu^*(e) = \begin{cases} \lambda & \text{if } e \geq e^*, \\ 0 & \text{if } e < e^*. \end{cases}$$

$$w^*(e) = \begin{cases} E(\theta) & \text{if } e \geq e^*, \\ \theta_L & \text{if } e < e^*. \end{cases}$$

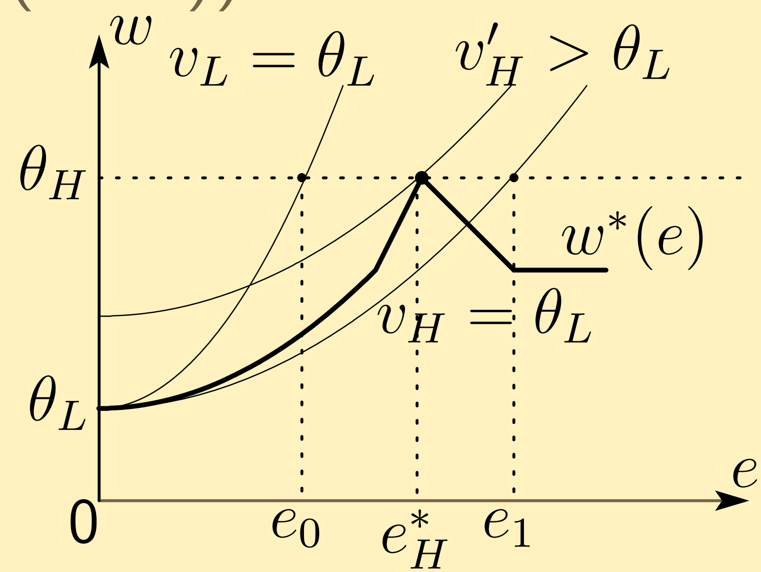


Intuitive criterion

Multiple equilibria and equilibrium refinement

Intuitive criterion (Cho and Kreps (1987)).

Consider a separating PBE in the figure. If type L chooses $e' \in (e_0, e^H)$, he/she will be worse off than choosing $e = 0$, regardless of the belief.



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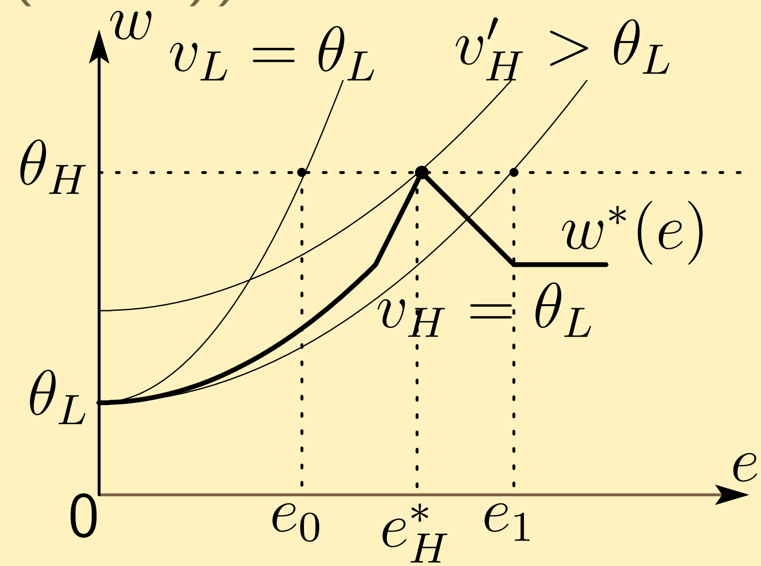

Type L will never choose $e > e_0$.

For $e \in (e_0, e_1)$, $\mu(e) \in [0, 1)$ is not reasonable.

So, $\mu(e) = 1$ for all $e \in (e_0, e_1)$.

Intuitive criterion

Multiple equilibria and equilibrium refinement

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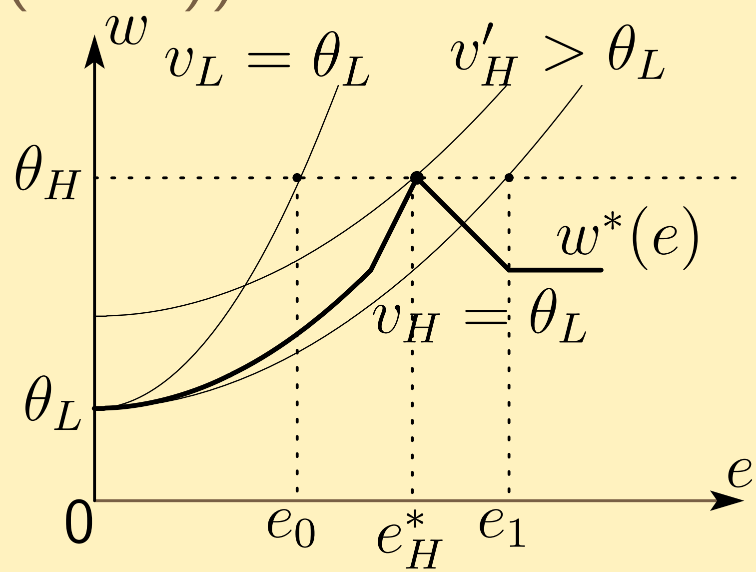

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So, $\mu(e) = 1$ for all $e \in (e_0, e_1)$.

Type H will be better of by choosing $e' \in (e_0, e_H^*)$.

Any separating PBE with $e_H^* > e_0$ is not sustained.

§13.D Screening

Basic assumptions As in Section 13.C,

1. Two types of workers θ_L and θ_H ($\theta_H > \theta_L > 0$), where $\lambda = \Pr(\theta = \theta_H) \in (0, 1)$.
2. The reservation wage of each worker is zero ($r(\theta_H) = r(\theta_L) = 0$).
3. Jobs might differ in the "task level," t, required of the worker.

   To simplify the analysis, assume that higher task levels add nothing to the output of the worker.

   The output of a type θ is θ regardless of the task.

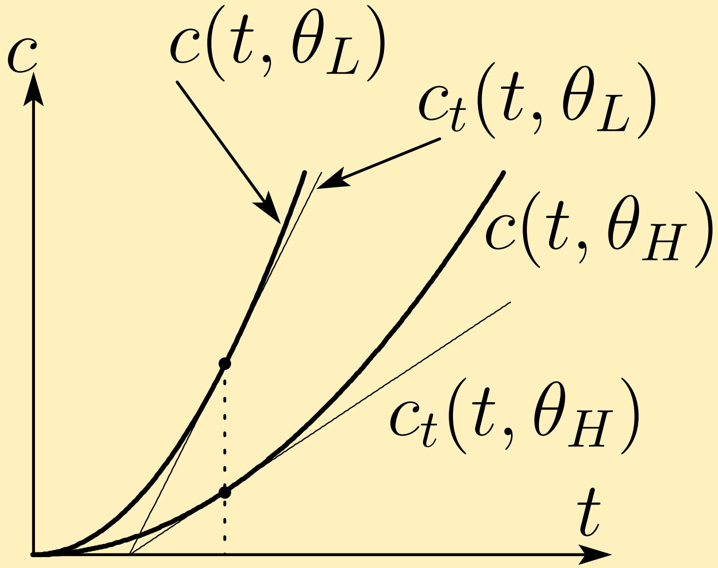
Basic assumptions (cont.)

$\forall\theta$, $c(0,\theta) = 0$.

$\forall\theta$, $\forall t$, $c_t(t,\theta) > 0$.

$\forall\theta$, $\forall t$, $c_{tt}(t,\theta) > 0$.

$\forall t$, $c(t,\theta_H) < c(t,\theta_L)$.



$\forall t$, $c_t(t,\theta_H) < c_t(t,\theta_L)$: single-crossing property.

Workers' payoff: $v(w,t|\theta) = w - c(t,\theta)$.

Screening

Timing We consider the following two stage game.

1. Two firms simultaneously announce sets of offered contracts. A contract is a pair (w, t).
2. Given the offers, each type worker chooses whether to accept a contract and, if so, which one.

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Proposition 13.D.1 In any SPNE of the screening game with **observable** worker types (complete information case), a type θ_i worker accepts contract $(w_i^*, t_i^*) = (\theta_i, 0)$, and firms earn zero profits.

Intuition: Bertrand competition between the firms.

Screening

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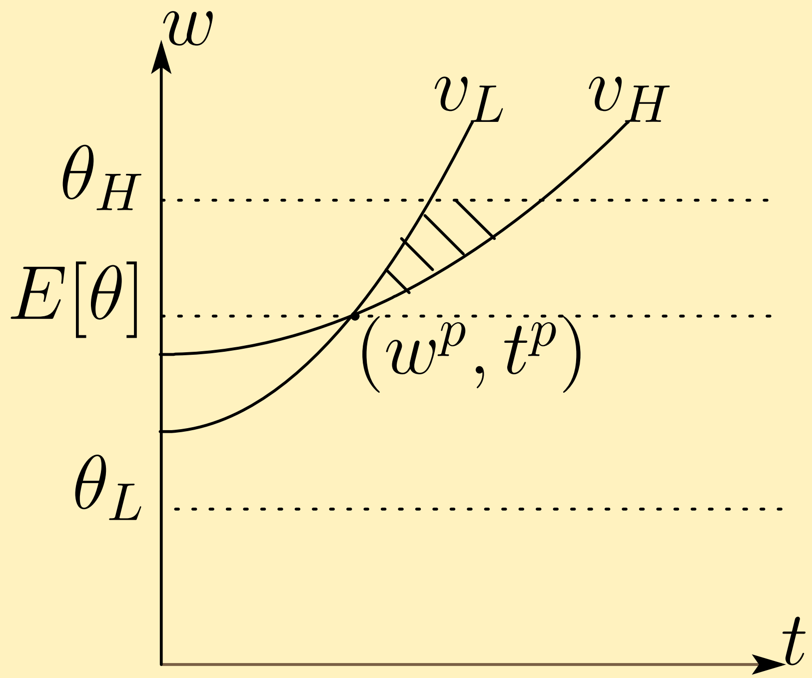
Lemma 13.D.2 No pooling equilibria exist.

Proof By contradiction. (w^p, t^p) is a pooling equilibrium contract. By Lemma 13.D.1, (w^p, t^p) lies on the break-even line (see Figure). Given the contract, a firm earns a positive profit by offering a contract on the shaded area.



(w_j, t_j): a contract signed by j workers ($j = H, L$).

Lemma 13.D.3 In a separating equilibrium, the contracts (w_H, t_H) and (w_L, t_L) yield zero profits. That is, $w_H = \theta_H$ and $w_L = \theta_L$.

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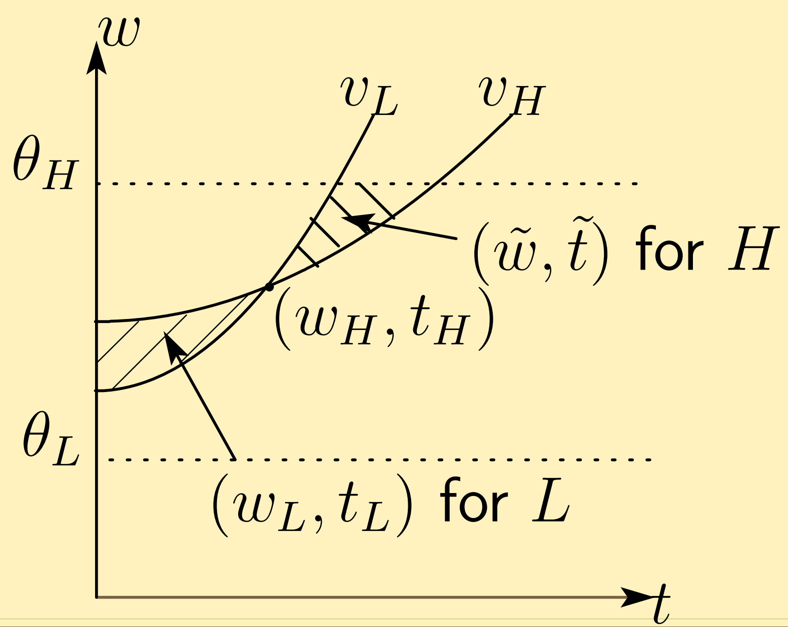
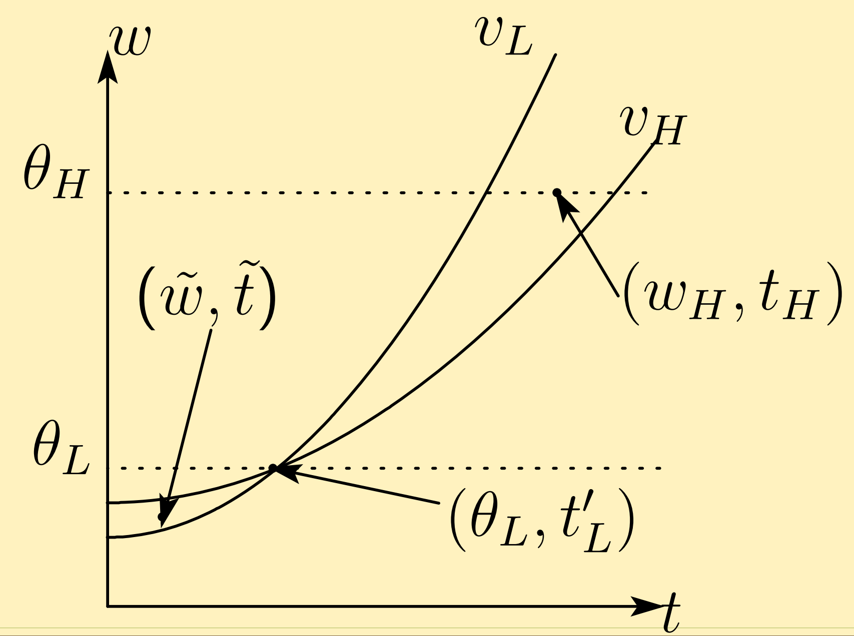
Lemma 13.D.4 In any separating equilibrium, L workers accept $(\theta_L, 0)$.





Screening

Lemma 13.D.5 In any separating equilibrium, H workers accept $(\theta_H, \hat{t}_H)$ such that

$$\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L).$$

$\hat{t}_H$ is determined not to induce type L to choose $(\theta_H, \hat{t}_H)$.
Type L's incentive: $\theta_H - c(\hat{t}_H, \theta_L) \leq \theta_L - c(0, \theta_L)$

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Intuition behind the lemma By Lemmas 13.D.3 and 13.D.4, $(w_L, t_L) = (\theta_L, 0)$ and $w_H = \theta_H$.

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t_H must be at least as large as $\hat{t}_H$ (no mimic).

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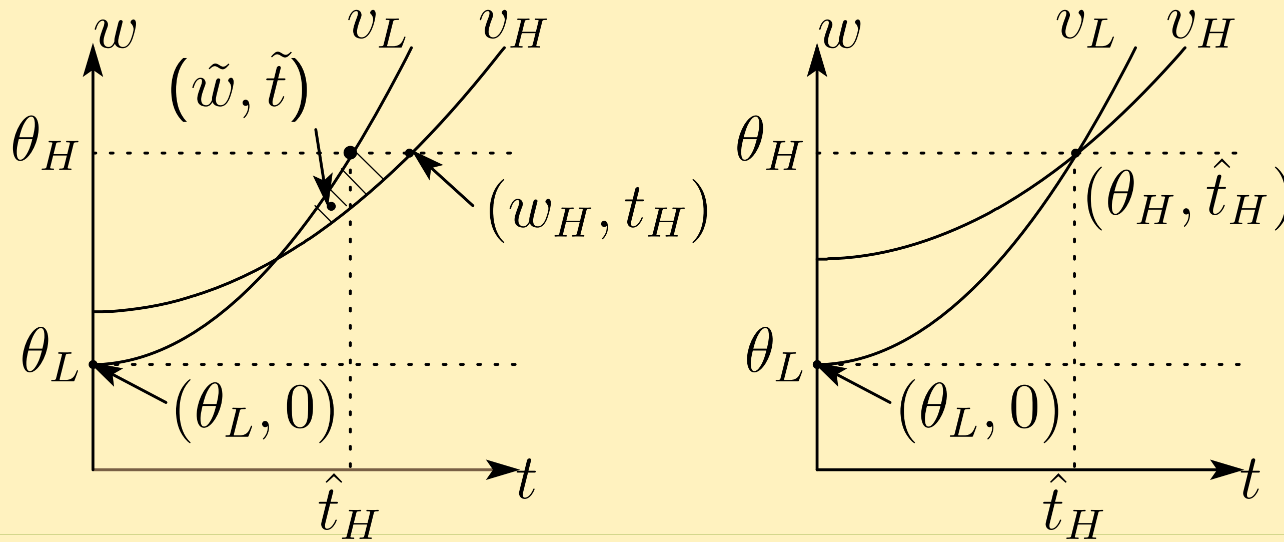
If $t_H > \hat{t}_H$, a firm can offer a more favorable contract for H workers.



Screening

Proposition 13.D.2 In any SPNE of the screening game, L workers accept $(\theta_L, 0)$, and H workers accept $(\theta_H, \hat{t}_H)$ such that $\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L)$.

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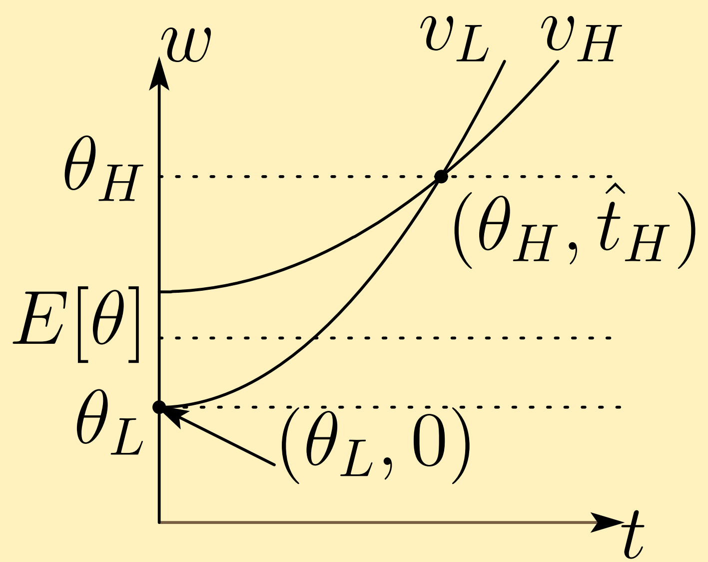
Remark This equilibrium is not always sustainable.



No pooling

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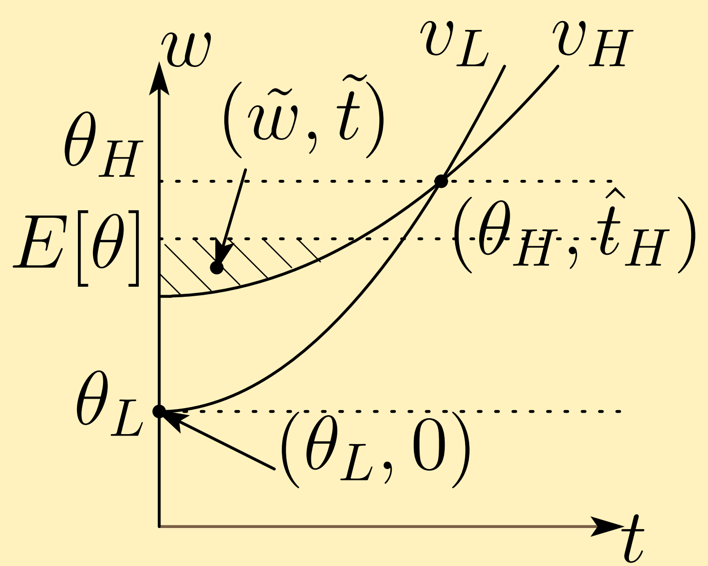
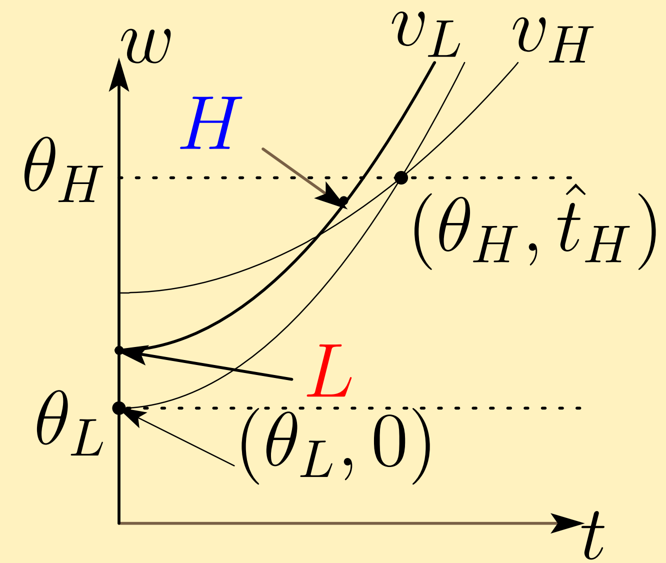


Deviation 1

Proposition 13.D.2 In any SPNE of the screening game, L workers accept $(\theta_L, 0)$, and H workers accept $(\theta_H, \hat{t}_H)$ such that $\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L)$.

Remark This equilibrium is not always sustainable.

Suppose that λ is large.
The gain of attracting type H is high.
Type H prefers menu H to $(\theta_H, \hat{t}_H)$.
But, type L also prefers menu H to $(\theta_L, 0)$.



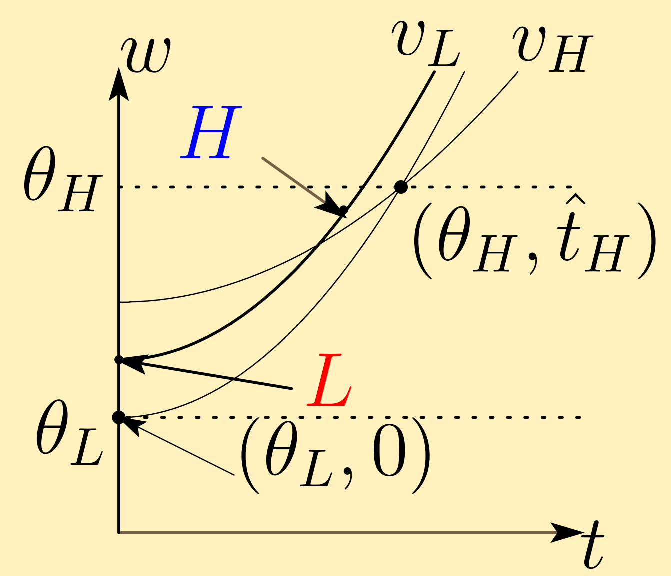
Deviation 2

Screening

Proposition 13.D.2 In any SPNE of the screening game, L workers accept $(\theta_L, 0)$, and H workers accept $(\theta_H, \hat{t}_H)$ such that $\theta_H - c(\hat{t}_H, \theta_L) = \theta_L - c(0, \theta_L)$.

Remark This equilibrium is not always sustainable.

Suppose that λ is large.
The gain of attracting type H is high.
Type H prefers menu H to $(\theta_H, \hat{t}_H)$.
Menu L is needed not to induce type L to choose menu H.
If the profit increase from menu H dominates the profit loss from menu L, the deviation can occur.



Deviation 2