Personalized pricing with heterogeneous mismatch costs*

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Abstract

Personalized pricing has become a reality through digitization. We examine firms' incentives to adopt one of the three pricing schemes: uniform, personalized, or group pricing in a Hotelling duopoly model. There are two types of consumer groups that are heterogeneous in their mismatch costs. We show that both firms employ personalized pricing in equilibrium regardless of the heterogeneity of consumer groups. If the consumer groups' heterogeneity is significant, the profits are higher when both firms use personalized pricing than when they employ uniform pricing; otherwise, the latter profits are higher than the former. Profits are highest when firms employ group pricing among the three cases. The ranking of consumer welfare among the three cases is opposite to that of profits.

Running head: Personalized pricing

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1 Introduction

Personalized pricing has become a reality through digitization (OECD, 2018).[1] In particular, mobile technology allows companies to offer discriminatory prices because customers have individual mobile devices to access online services (Esteves and Resende, 2016). A famous example of such personalized pricing is Uber's "route-based pricing," which depends on customers' willingness to pay (Bloomberg, 2017).[2] Although personalized pricing can help firms capture each consumer's willingness to pay, the strategic interaction with rivals can also exert opposite effects. This possibility raises questions about whether personalized pricing can improve a firm's profits and benefit the consumers.

Regarding the influence of personalized pricing on firms' profits, there has been broad discussion since the seminal paper by Thisse and Vives (1988). They concluded that personalized pricing induces a prisoner's dilemma where all firms become worse off than the situation when all the firms commit to uniform pricing (Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Zhang, 2011). However, several studies show that personalized pricing does not necessarily result in a prisoner's dilemma in the case of firm asymmetry (e.g., a quality difference (Shaffer and Zhang, 2002), quality choice (Ghose and Huang, 2009), and initial cost difference with R&D (Matsumura and Matsushima, 2015)). Therefore, discussions on the effect of personalized pricing on firms' profits are inconclusive.

This study examines consumer heterogeneity and develops a linear market model comprising two consumer groups—one incurring high transportation costs (high-end consumers) and the other incurring low transportation costs (low-end consumers). The model also comprises two symmetric firms competing in this market and choosing one of the pricing schemes: uniform, personalized, or group pricing.

The results are as follows. Both firms always employ personalized pricing in equilibrium, which enhances both firms' profits compared with the situation where both employ uniform pricing if the consumer groups are sufficiently heterogeneous. Furthermore, firms' profits when they both employ group pricing are always the highest compared with the other two cases mentioned before. We show that the consumer surplus in group pricing is always the lowest when comparing the aforementioned three cases. The consumer surplus in uniform pricing is the highest if the consumer groups are sufficiently heterogeneous.

In this regard, the intuition stems from how the low-end consumers influence the uniform prices of firms. When the low-end consumers incur a sufficiently low transportation cost (relative to the high-end consumers), they are price-sensitive. When firms adopt a uniform pricing scheme, they offer sufficiently low prices to capture the price-elastic consumers; this diminishes their profits from high-end consumers. However, when both firms adopt personalized pricing, they customize each consumer's price and delink the markets for high- and low-end consumers. This makes the competition mode similar to an asymmetric Bertrand competition for each consumer. Unlike a uniform pricing scheme, the adoption of personalized pricing intensifies the competition for low-end consumers (a negative effect). Nevertheless, it enables each firm to set higher prices for high-end consumers located closer to that firm (a positive effect). In summary, personalized pricing triggers market segmentation such that low-end consumers pay less. In contrast, some high-end consumers pay more, which benefits the firms when the heterogeneity between the two consumer groups is sufficiently significant.

Group pricing is the simplest way to delink the two markets for the two consumer types and allows firms to escape fierce Bertrand competition under personalized pricing at each point. However, group pricing is not attainable in equilibrium. The comparison between group and personalized pricing is equivalent to that between uniform and personalized pricing in Thisse and Vives (1988) when we focus on each consumer group. We can directly apply the conclusion in Thisse and Vives (1988) and find that each firm has an incentive to employ personalized pricing, given that the rival employs group pricing. Therefore, group pricing is not sustainable as an equilibrium outcome, although firms are better off if they employ group pricing.

In a recent study, Esteves (2022) considers a static setting where personalized pricing can become a winning strategy for competing firms.[3] The intuition relies on the heterogeneity in the quantity demanded by consumers. As in Shin and Sudhir (2010), some consumers demand more products than others. We capture consumers' heterogeneity by their horizontal valuation of products (i.e., the transportation cost) instead of the quantity purchased. We also consider the endogenous choices of pricing schemes. Given these differences, our study complements Esteves (2022).

Esteves and Shuai (2022) also discuss the profitability of personalized pricing in a Hotelling duopoly with a CES type elastic demand (Gu and Wenzel, 2009).[4] They show that personalized pricing is more profitable than uniform pricing if the elasticity is high. The reason is that personalized prices are independent of the elasticity, although uniform prices decrease with the elasticity. Contrasting to their paper, we consider the endogenous choices of pricing schemes in addition to the profit comparison. Also, our paper differs from theirs regarding the relationship between price elasticities and personalized prices. Given these differences, our study complements Esteves and Shuai (2022).

Rhodes and Zhou (2022) investigate the effects of personalized pricing on profits and welfare, using a generalized oligopoly model based on Perloff and Salop (1985).[5] The authors generalize the number of firms and the degree of market coverage and show that in the short run (e.g., when firms' number is exogenously given), personalized pricing would benefit firms and harm consumers compared with uniform pricing, only when the market coverage is low. By incorporating consumers' heterogeneity in their mismatch costs, we show that the same argument could hold even under a *full* market coverage. Therefore, we provide a complement to Rhodes and Zhou (2022).

The remainder of this paper is organized as follows. Section 2 formulates the model.

Then, section 3 analyzes the model and presents the main results. Finally, section 4 concludes the paper. The detailed mathematical procedures are available in the Online Appendix.

2 Model

We consider a product characteristic space with interval $[0, 1]$. There are two consumer types called consumer k $(= L, H)$. We refer to the markets with consumers H and L as markets H and L. We normalize the number of consumers to 1 and assume that the shares of consumers H and L are λ and $1 - \lambda$, respectively. The distribution of consumers in each market is uniform on the interval $[0, 1]$.

There are two firms—firms 0 and 1 located at 0 and 1, respectively. We assume that firm i can choose one of the three pricing schemes—uniform, personalized, or group pricing. In uniform pricing, the firm offers the same price to all consumers regardless of their location or type. In personalized pricing, the firm can customize the price for each consumer located at $x \in [0, 1]$ in market k. Finally, in group pricing, the firm offers discriminatory prices to consumers H and L.

The utility function for a consumer located at $x \in [0, 1]$ in market k is

$$U(x, k) = \begin{cases} v - p_0(x, k) - t_k x & \text{buying from firm 0,} \\ v - p_1(x, k) - t_k(1 - x) & \text{buying from firm 1,} \end{cases}$$

where v is the willingness to pay for the ideal product, $p_i(x, k)$ is the customized price offered to consumers located at x in market k, and t_k is the per-distance transportation (mismatch) cost in market k, as in Armstrong (2006, pp.116-117). Note that, for any $x \in [0, 1]$ and $k = H, L$, $p_i(x, k)$ is constant when firm i uses uniform pricing. We denote the uniform price offered by firm i by p_i. When firm i uses group pricing, it offers a uniform price $p_i(k)$ to market k. Assume $t_H > t_L > 0$ and define $\tau \equiv t_L/t_H$. We perform comparative statics on τ keeping t_H fixed throughout the study.

Our model effectively captures a feature of app-based taxi services, which is price discrimination based on users' characteristics. Some firms can offer group pricing based on users' mobile phone types, such as iPhones versus other mobile phones.[6] Users of iPhones tend to be less price elastic because of their higher wealth levels compared to users of other mobile phones.[7] This difference in price elasticity can be captured by heterogeneity of transportation costs, which represent price elasticity. Additionally, personalized pricing can be implemented by considering consumers' preferences for taxi services, such as their urgency or willingness to consider alternative transportation options, as exemplified by Uber's route-based pricing. To capture these preferences, we use locations on the interval $[0, 1]$.

Each firm produces a product without cost. Then, firm i's profit is as follows:

$$\pi_0[p_0(x,k), p_1(x,k)] \equiv \lambda \int_0^{x_H} p_0(x,H)dx + (1-\lambda)\int_0^{x_L} p_0(x,L)dx,$$
$$\pi_1[p_0(x,k), p_1(x,k)] \equiv \lambda \int_{x_H}^1 p_1(x,H)dx + (1-\lambda)\int_{x_L}^1 p_1(x,L)dx,$$

where x_k is the location of indifferent consumers in market k.

We follow the timing structure of Thisse and Vives (1988). In the first stage, each firm selects one of the three pricing schemes. In the second stage, a firm employing a uniform or group pricing offers its observable uniform price. Subsequently, a firm employing personalized pricing offers personalized prices, $p_i(x,k)$, determined by the consumer types and locations. If the two firms adopt the same pricing scheme, they simultaneously determine their prices. The timing of pricing offers are in line with those considered in the literature (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 2002; Clavorà Braulin and Valletti, 2016; Choe et al., 2018; Chen et al., 2020; Esteves, 2022). Using backward induction, we solve this game.

3 Analysis

We need to solve six types of subgames: (i) Both firms choose uniform pricing; (ii) Both firms choose personalized pricing; (iii) Both firms choose group pricing; (iv) One firm chooses uniform pricing, and the other one chooses personalized pricing; (v) One firm chooses group pricing, and the other one chooses uniform pricing, and (vi) One firm chooses group pricing, and the other one chooses personalized pricing. As the calculations are simple, we provide only the results in the subgames.

3.1 Both firms choose uniform pricing

We consider the case where both firms employ uniform pricing. They simultaneously offer their uniform prices. We use the superscript "UU" to denote this case.

We classify the outcome in this subgame into two cases: (1) a pure strategy equilibrium exists; (2) no pure strategy equilibrium exists. The former case appears if τ satisfies the following inequality, otherwise the latter appears:

$$\tau \geq \frac{-(1-\lambda)(2+\lambda)+2\sqrt{1-\lambda}}{\lambda(3+\lambda)} \equiv \tau_{uu}. \tag{1}$$

In the first case ($\tau \geq \tau_{uu}$), we obtain each firm's price and profit as follows:

$$p \equiv \frac{t_H \tau}{1-\lambda(1-\tau)}, \quad \pi \equiv \frac{t_H \tau}{2-2\lambda(1-\tau)}. \tag{2}$$

In the second case (when $\tau < \tau_{uu}$), we instead consider a mixed strategy equilibrium where, (i) firm 0 chooses a low price p_{0l} with probability α and a high price p_{0h} with probability $1-\alpha$, and (ii) firm 1 chooses a low price p_{1l} with probability β and a high price p_{1h} with probability $1-\beta$. We use the superscript m to indicate an outcome in a mixed strategy

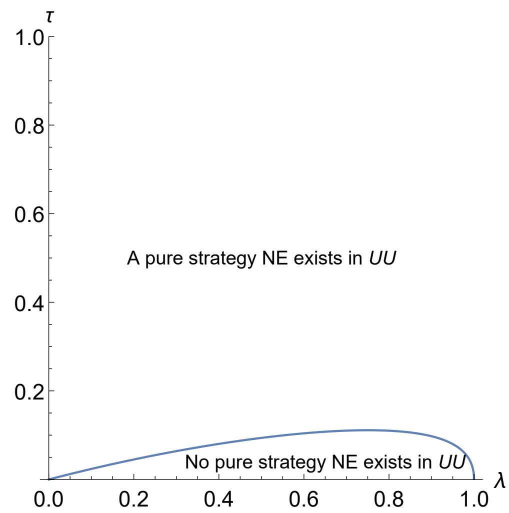


Figure 1: The area in which a pure strategy NE exists in UU

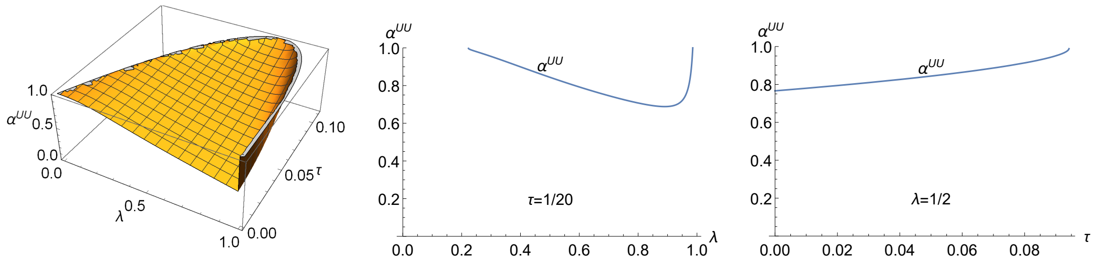
equilibrium if necessary.

We focus on a symmetric mixed strategy NE such that $x_L(p_{0h}, p_{1l}) < 0$, $x_L(p_{0l}, p_{1h}) > 1$, $x_L(p_{0l}, p_{1l}) = x_L(p_{0h}, p_{1h}) = 1/2$, $x_H(p_{0l}, p_{1h}) \in (0, 1)$, $x_H(p_{0h}, p_{1l}) \in (0, 1)$, $x_H(p_{0l}, p_{1l}) = x_H(p_{0h}, p_{1h}) = 1/2$, and $\alpha = \beta \in (0, 1)$. In this equilibrium, the resulting outcome in market L is the corner solution in which the low-price firm obtains all type L consumers' demand if the rival offers the high price. We can numerically obtain a symmetric mixed strategy equilibrium with p_{il}^{UUm}, p_{ih}^{UUm}, and $\alpha^{UU} = \beta^{UU} \in (0, 1)$ that satisfy (the detail is available in (A.1) in the Appendix):

$$\begin{aligned} & \text{The expected profit of firm } i \text{ when it offers } p_{il}^{UUm} \\ = \; & \text{The expected profit of firm } i \text{ when it offers } p_{ih}^{UUm} = \pi_i^{UUm}. \end{aligned}$$

Given the complexity of the last two simultaneous equations, we cannot explicitly derive the values of α^{UU} and β^{UU} but numerically show those values in Figure 2. The region where $\alpha^{UU}(\lambda, \tau) \in (0, 1)$ coincides with that of the pure strategy equilibrium is not sustainable.

The relationship between α^{UU} and λ can be positive and negative, depending on λ (see $\alpha^{UU}(\lambda)$ in Figure 2): it is negative if λ takes on an intermediate value; it is positive if λ is



$\alpha^{UU}(\lambda, \tau)$ $\alpha^{UU}(\lambda)$ $(\tau = 1/20)$ $\alpha^{UU}(\tau)$ $(\lambda = 1/2)$

Figure 2: Mixed strategy in which both firms employ uniform pricing

large enough but is not near 1. When λ is near zero, the firms offer a low uniform price, which is close to τt_H (see (2)), with probability 1, resulting in a pure strategy equilibrium. However, when λ takes on an intermediate value, a pure strategy equilibrium is not sustainable due to the incentive for firms to deviate by offering higher prices to achieve higher price cost margin. To eliminate this deviation incentive, firms need to adopt a mixed strategy involving probabilistic offers of high prices, thereby making low prices more profitable through demand expansion of type L consumers in the case where the rival offers such a high price. As λ increases, the deviation incentive becomes stronger because the gain from type H consumers increases. The increased gain induces firms to offer high prices with higher probabilities (a decrease of α^{UU}).

In the second case where λ is large enough but not near 1, the importance of supplying to type L consumers is low enough, inducing firms to increase p_{il}^{UUm} along with an increase of α^{UU} to target type H consumers. However, when λ is near 1, the firms mainly focus on type H consumers and offer p, close to t_H, in (2) with probability 1.

We can summarize the discussion as follows:

$$p_i^{UU} = \begin{cases} \dfrac{t_H\tau}{1-\lambda(1-\tau)} & \text{if } \tau \geq \tau_{uu}, \\ \begin{cases} p_{il}^{UUm} \text{ in (A.1) with probability } \alpha^{UU}\ (\beta^{UU}), \\ p_{ih}^{UUm} \text{ in (A.1) with probability } 1-\alpha^{UU}\ (1-\beta^{UU}). \end{cases} & \text{if } \tau < \tau_{uu}, \end{cases} \tag{3}$$

$$\pi_i^{UU} = \begin{cases} \dfrac{t_H\tau}{2-2\lambda(1-\tau)} & \text{if } \tau \geq \tau_{uu}, \\ \pi_i^{UUm} \text{ in (A.1)}, & \text{if } \tau < \tau_{uu}. \end{cases} \tag{4}$$

Conducting comparative statics, we find that when λ is large and τ is smaller than but sufficiently close to τ_{uu}, a decrease in τ enlarges the profits of the two firms. If $\tau < \tau_{uu}$, each firm sets a high price, p_{ih}^{UUm}, with a positive probability to escape from fierce competition in market L and to concentrate on market H (see $\alpha^{UUm}(\lambda,\tau)$ in Figure 2). The strategic effect mitigates price competition, although the direct effect of a decrease in τ intensifies price competition. If τ is smaller than but sufficiently close to τ_{uu}, the marginal impact of the strategic effect is larger than the direct effect because the probability of offering p_{ih}^{UUm} becomes strictly positive, implying that the strategic effect begins to be effective and has a first-order impact on competition. Otherwise, the direct effect of a decrease in τ dominates the strategic effect, aggravating competition. We summarize the above finding in the following proposition:

Proposition 1 *Suppose that both firms employ uniform pricing. When τ is smaller than but sufficiently close to τ_{uu} and λ is larger than a threshold value, a decrease in τ increases the expected profits of the firms.*

3.2 Both firms choose personalized pricing

We consider the case where both the firms use personalized pricing. They simultaneously offer personalized prices for distinct consumers. We use the superscript "*PP*" to denote this case.

In each market, the firms face an asymmetric Bertrand competition at each location, $x \in [0, 1]$. The firms choose the following prices at location x in market k $(= H, L)$.

$$p_0^{PP}(x,k) \equiv \begin{cases} t_k(1-x) - t_k x & \text{for } x \in [0, 1/2], \\ 0 & \text{for } x \in (1/2, 1], \end{cases}$$
$$p_1^{PP}(x,k) \equiv \begin{cases} 0 & \text{for } x \in [0, 1/2], \\ t_k x - t_k(1-x) & \text{for } x \in (1/2, 1]. \end{cases}$$

Then, consumers in $[0, 1/2]$ and $(1/2, 1]$ buy from firms 0 and 1, respectively. We obtain the profit of each firm as follows:

$$\pi^{PP} \equiv \frac{t_H(\lambda + \tau(1-\lambda))}{4}. \tag{5}$$

When $\lambda = 1$ or $\tau = 1$, the outcome is the same as that with the personalized pricing in Thisse and Vives (1988). When $\tau \neq 1$, π^{PP} decreases when λ decreases. However, $p_i^{PP}(x,k)$ $(k = H, L)$ remains the same because the two markets are independent. Similarly, when $\lambda \neq 1$, π^{PP} and $p_i^{PP}(x,L)$ decrease when τ decreases, and $p_i^{PP}(x,H)$ remains the same.

3.3 Both firms choose group pricing

When both firms adopt group pricing, they offer uniform prices to each consumer group. Therefore, they compete in markets H and L, respectively. We use the superscript "GG" to denote this case.

The prices and the resulting profits are

$$p^{GG}(H) \equiv t_H,\ p^{GG}(L) \equiv t_L = \tau t_H, \quad \pi^{GG} \equiv \frac{t_H(\lambda + (1-\lambda)\tau)}{2}. \tag{6}$$

A decrease in λ and a decrease in τ lower profits.

3.4 One firm chooses uniform pricing, and the other one chooses personalized pricing

We consider the case where one firm (say firm 0) employs uniform pricing and the other (say firm 1) adopts personalized pricing. In the second stage, firms 0 and 1 become the leader and the follower, respectively. We use the superscript "UP" to denote this case.

We check the optimal pricing of the firms. Since firm 1 becomes the follower, given p_0, firm 1 chooses $p_1^{UP}(x, k) = \max\{p_0 + t_k x - t_k(1 - x), 0\}$ at $x \in [0, 1]$ in market k. Next, we consider firm 0's decision. A consumer who is indifferent between buying from firm 0 or firm 1 must satisfy $p_0 + t_k x = p_1 + t_k(1 - x)$, from which we have $p_1(x, t_k) = p_0 + (2x - 1)t_k$, which must be non-negative. By solving $p_0 + (2x_k^{UP} - 1)t_k = 0$, for x_k^{UP}, we obtain the location of the indifferent consumer in market k, $x_k^{UP}(p_0) \equiv (t_k - p_0)/(2t_k)$. Since $x_k^{UP}(p_0) \in [0, 1]$, firm 0's profit function is

$$\pi_0^{UP}(p_0) \equiv \begin{cases} p_0\{\lambda x_H^{UP}(p_0) + (1 - \lambda)x_L^{UP}(p_0)\} & \text{if} \quad 0 \le p_0 \le \tau t_H, \\ p_0 \lambda x_H^{UP}(p_0) & \text{if} \quad \tau t_H \le p_0 \le t_H, \\ 0 & \text{if} \quad t_H \le p_0. \end{cases}$$

By solving the profit maximization problem of firm 0, we obtain the following optimal price:

$$p_0^{UP} = \begin{cases} \dfrac{t_H \tau}{2 - 2\lambda(1 - \tau)} & \text{if} \quad 0 < \lambda \le \dfrac{\tau}{1 - \tau} \quad \text{or} \quad 1/2 \le \tau, \\ \dfrac{t_H}{2} & \text{if} \quad \dfrac{\tau}{1 - \tau} \le \lambda < 1 \quad \text{and} \quad \tau < 1/2. \end{cases}$$

If λ is large and τ is small, firm 0 will abandon the supply to type L consumers, and firm 1 will monopolize the market for type L consumers. Otherwise, firm 0 will serve both consumer types by offering p_0^{UP}, which is lower than $t_H/2$ unless $\tau = 1$. This pricing of firm 0 implies intense price competition when the ratio of type H consumers is low (λ is small).

Using this result, we obtain the profit of firm 0:

$$\pi_0^{UP} = \begin{cases} \dfrac{t_H\tau}{8-8\lambda(1-\tau)} & \text{if } \ 0<\lambda\le\dfrac{\tau}{1-\tau} \ \text{ or } \ 1/2\le\tau, \\ \dfrac{t_H\lambda}{8} & \text{if } \ \dfrac{\tau}{1-\tau}\le\lambda<1 \ \text{ and } \ \tau<1/2. \end{cases} \tag{7}$$

We obtain the profit of firm 1 is as follows:

$$\pi_1^{UP} = \begin{cases} \dfrac{t_H[9\tau+4\lambda(1-\tau)^2-4\lambda^2(1-\tau)^2]}{16-16\lambda(1-\tau)} & \text{if } \ 0<\lambda\le\dfrac{\tau}{1-\tau} \ \text{ or } \ 1/2\le\tau, \\ \dfrac{t_H(8+\lambda)}{16} & \text{if } \ \dfrac{\tau}{1-\tau}\le\lambda<1 \ \text{ and } \ \tau<1/2. \end{cases} \tag{8}$$

A decrease in λ and a decrease in τ lower profits.

3.5 One firm chooses group pricing, and the other one chooses uniform pricing

We consider the case where one firm (say firm 0) employs group pricing and offers $p_0(k)$ to market k and the other (say firm 1) adopts uniform pricing and offers p_1. They simultaneously offer their prices. We use the superscript "GU" to denote this case. The indifferent consumer in market k is denoted by $x_k^{GU}(p_0(k), p_1) = (-p_0(k) + p_1 + t_k)/(2t_k)$.

We need to classify the outcome in this subgame into the following two cases: (1) a pure strategy equilibrium exists; (2) no pure strategy equilibrium exists. The former case appears if τ satisfies the following inequality, otherwise the latter appears:

$$\tau \ge \frac{-8+3\lambda+9\lambda^2+4\sqrt{4-3\lambda}}{3\lambda(5+3\lambda)} \equiv \tau_{gu}. \tag{9}$$

In the first case ($\tau \ge \tau_{gu}$), firms solves $\max_{p_0(H),p_0(L)} \ \lambda p_0(H)x_H^{GU}(p_0(H), p_1) + (1-\lambda)p_0(L)x_L^{GU}(p_0(L), p_1)$ and $\max_{p_1} \ p_1\{\lambda(1-x_H^{GU}(p_0(H), p_1)) + (1-\lambda)(1-x_L^{GU}(p_0(L), p_1))\}$,

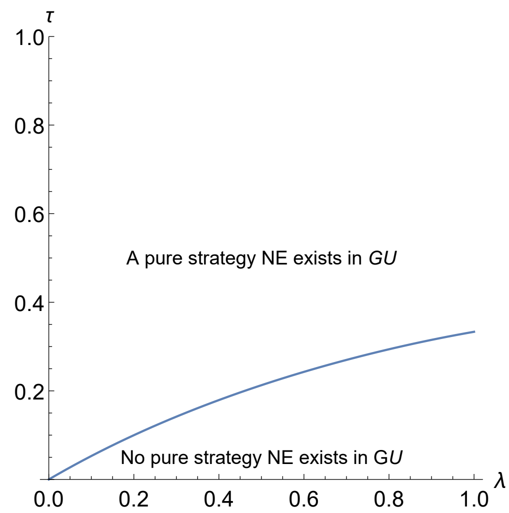


Figure 3: The area in which a pure strategy NE exists in GU

from which we have

$$\begin{aligned} &p_0^{GU*}(H) \equiv \frac{t_H(1-\lambda+\tau+\lambda\tau)}{2(1-\lambda+\lambda\tau)},\ p_0^{GU*}(L) \equiv \frac{t_H(2-\lambda+\lambda\tau)}{2(1-\lambda+\lambda\tau)},\ p_1^{GU*} \equiv \frac{t_H\tau}{1-\lambda+\lambda\tau} \\ &\pi_0^{GU*} \equiv \frac{t_H\lambda(1-\lambda+\tau+\lambda\tau)^2}{8(1-\lambda+\lambda\tau)^2} + \frac{t_H(1-\lambda)\tau(2-\lambda+\lambda\tau)^2}{8(1-\lambda+\lambda\tau)^2)},\ \pi_1^{GU*} \equiv \frac{t_H\tau}{2(1-\lambda+\lambda\tau)}. \end{aligned} \tag{10}$$

In the second case ($\tau < \tau_{gu}$), we instead consider a mixed strategy equilibrium where, (i) firm 0 employs a pure strategy and offers $p_0(H)$ and $p_0(L)$, and (ii) firm 1 chooses a low price p_{1l} with probability β and a high price p_{1h} with probability $1-\beta$.

We focus on a mixed strategy NE such that $x_L^{GU}(p_0(L), p_{1l}) \in (0,1)$, $x_L^{GU}(p_0(L), p_{1h}) > 1$, $x_H^{GU}(p_0(H), p_{1l}) \in (0,1)$, $x_H^{GU}(p_0(H), p_{1h}) \in (0,1)$, and $\beta \in (0,1)$. In this mixed strategy equilibrium, the resulting outcome in market L is the corner solution where firm 0 obtains all type L consumers if the rival offers the high price. We can obtain a mixed strategy NE with $p_0^m(H)$, $p_0^m(L)$, p_{1l}^m, p_{1h}^m, and $\beta^{GU} \in (0,1)$ that satisfies (the detail is available in (A.2) in the

Appendix):

$$\begin{aligned} & \text{The expected profit of firm 1 when it offers } p_{1l}^{m} \\ = \; & \text{The expected profit of firm 1 when it offers } p_{ih}^{m} = \pi_1^{GUm}. \end{aligned}$$

Although we can explicitly derive the value of β which is complex, we numerically show this value in Figure 4. The region where $\beta^{GU}(\lambda,\tau) \in (0,1)$ coincides with that of the pure strategy equilibrium is not sustainable.

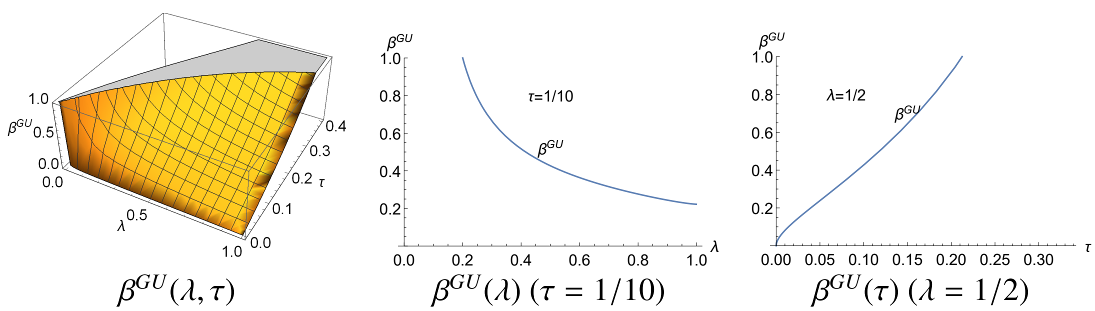


Figure 4: Mixed strategy in which one firm chooses group pricing and the other chooses uniform pricing

The relationship between β^{GU} and λ is always negative (see $\beta^{GU}(\lambda)$ in Figure 4). We can apply the intuition in the case of UU here. Given the group pricing by firm 0, firm 1 deviates by raising its price because supplying only to type H consumers is profitable. To eliminate this deviation incentive, $p_0(H)$ should be lower and firm 1 needs to induce firm 0 to set such a low $p_0(H)$ by offering a low price with probability β^{GU}. However, as λ increases, firm 1's incentive to offer a high price becomes stronger because the gain from type H consumers increases. The increased gain induces firm 1 to offer a high price with higher probabilities (a decrease of β^{GU}). Because no pure strategy exists for any high values of λ if $\tau < \tau_{gu}$, the negative relationship remains.

We can summarize the discussion as follows:

$$p_0^{GU}(H) = \begin{cases} \dfrac{t_H(1+\tau-(1-\tau)\lambda)}{2(1-\lambda(1-\tau))} & \text{if } \tau \geq \tau_{gu}, \\ p_0^m(H) \text{ in (A.2)} & \text{if } \tau < \tau_{gu}, \end{cases} \tag{11}$$

$$p_0^{GU}(L) = \begin{cases} \dfrac{t_H(2-(1-\tau)\lambda)}{2(1-\lambda(1-\tau))} & \text{if } \tau \geq \tau_{gu}, \\ p_0^m(L) \text{ in (A.2)} & \text{if } \tau < \tau_{gu}, \end{cases} \tag{12}$$

$$p_1^{GU} = \begin{cases} \dfrac{t_H\tau}{1-\lambda(1-\tau)} & \text{if } \tau \geq \tau_{gu}, \\ \begin{cases} p_{il}^m \text{ in (A.2) with probability } \beta^{GU}, \\ p_{ih}^m \text{ in (A.2) with probability } 1-\beta^{GU}, \end{cases} & \text{if } \tau < \tau_{gu}, \end{cases} \tag{13}$$

$$\pi_0^{GU} = \begin{cases} \dfrac{t_H\lambda(1-\lambda+\tau+\lambda\tau)^2}{8(1-\ \lambda+\lambda\tau)^2} + \dfrac{(t_H(1-\lambda)\tau(2-\lambda+\lambda\tau)^2}{8(1-\lambda+\lambda\tau)^2} & \text{if } \tau \geq \tau_{gu}, \\ \pi_0^{GUm} \equiv p_0^m(H)\lambda(\beta^m x_H(p_0^m(H), p_{il}^m) + (1-\beta^m)x_H(p_0^m(H), p_{ih}^m)) & \\ \qquad +p_0^m(L)(1-\lambda)(\beta^m x_L(p_0^m(L), p_{il}^m) + (1-\beta^m)), & \text{if } \tau < \tau_{gu}. \end{cases} \tag{14}$$

$$\pi_1^{GU} = \begin{cases} \dfrac{t_H\tau}{2-2\lambda(1-\tau)} & \text{if } \tau \geq \tau_{gu}, \\ \pi_1^{GUm} \text{ in (A.2),} & \text{if } \tau < \tau_{gu}. \end{cases} \tag{15}$$

We find that when $\tau < \tau_{gu}$, a decrease in τ *increases* firm 1's expected uniform price, mitigating price competition. A decrease in τ has direct and strategic effects on profits. First, a decrease in τ directly intensifies competition in market L, which harms firm 1. Second, when $\tau < \tau_{gu}$, the lower τ, the lower the probability of setting p_{1l}, β^{GU} (see Figure 4), because the market for type L consumers becomes less profitable. The change of β^{GU} induces firm 0 to offer a higher price in market H because of the strategic complementarity of both firms' prices, which benefits firm 1. The relative significance of the two contrasting effects depends on the pricing scheme of firm 0. When firm 0 adopts group pricing, firm 0 can offer discriminatory prices in the two markets, implying that firm 0 can keep its price in market H high for any τ (see $p_0^m(H)$ in (A.2)). Such a high price of firm 0 in market H allows firm 1 to concentrate on market H to obtain a positive profit even if τ is sufficiently small (see β^{GU} in

Figure 4). Therefore, for firm 1's profit, the strategic effect dominates the direct effect.

However, when τ is sufficiently small, the mitigation of price competition becomes weak. Firm 0 is less likely to compete with firm 1 in market L because of a low β^{GU}, weakening the strategic complement effect on $p_0^m(L)$. In fact, $p_0^m(L)$ increases in τ if τ is sufficiently small; otherwise, $p_0^m(L)$ decreases in τ. Therefore, when τ is smaller than $\bar{\tau}$ defined in Figure 5, a reduction in τ reduces $p_0^m(L)$ and such a price reduction consequently causes a profit reduction for firm 0, which captures the entire demand in market L with probability $1-\beta^{GU}$. We summarize the above finding in the following proposition:

Proposition 2 *Suppose that one firm (firm 0) employs group pricing and the other one (firm 1) employs uniform pricing. When $\bar{\tau} < \tau < \tau_{gu}$, a decrease in τ increases the expected profit of firm 0. When $\tau < \tau_{gu}$, a decrease in τ increases the expected profit of firm 1.*

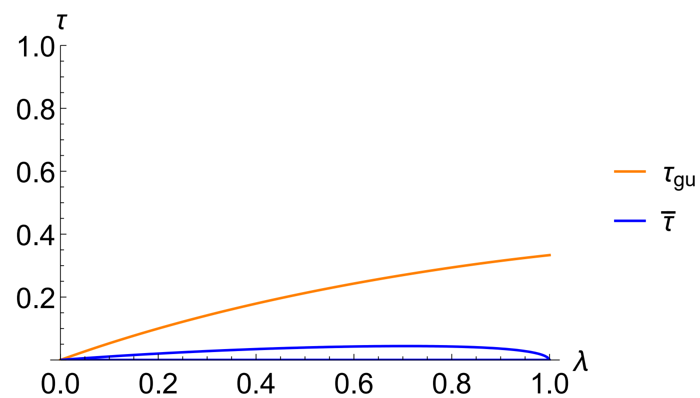


Figure 5: The area in which a decrease in τ increases profits

3.6 One firm chooses group pricing, and the other chooses personalized pricing

We consider firm 0 employs group pricing and firm 1 employs personalized pricing. In the second stage, firms 0 and 1 become the leader and the follower, respectively. We use the superscript "GP" to denote this case. Given firm 0's price $p_0(k)$, firm 1 offers $p_1 = p_0(k) + (2x - 1)t_k$. The indifferent consumer in each market is denoted by $x_k^{GP}(p_0(k)) \equiv (t_k - p_0(k))/(2t_k)$. Firm 0 then solves $\max_{p_0(k)} x_k^{GP}(p_0(k))p_0(k)$, from which we have

$$p_0^{GP}(k) \equiv \frac{t_k}{2},\ \pi_0^{GP} \equiv \frac{t_H((1-\lambda)\tau+\lambda)}{8},\ \pi_1^{GP} \equiv \frac{9t_H((1-\lambda)\tau+\lambda)}{16}. \tag{16}$$

Each market outcome is the same as in Thisse and Vives (1988) except for the market size. As in the previous arguments, a decrease in λ and a decrease in τ lower profits.

4 Equilibrium pricing scheme

First, we derive the equilibrium pricing schemes in the first stage and show that both firms employ personalized pricing in equilibrium. Second, we discuss how pricing schemes influence profits and welfare.

4.1 Decisions on pricing schemes

The discussions in the previous subsections provide the payoff matrix in the first-stage game.

Firm 0/Firm 1	uniform	personalized	group
uniform	π^{UU} in (4)	π_1^{UP} in (8)	π_0^{GU} in (14)
	π^{UU} in (4)	π_0^{UP} in (7)	π_1^{GU} in (15)
personalized	π_0^{UP} in (7)	π^{PP} in (5)	π_0^{GP} in (16)
	π_1^{UP} in (8)	π^{PP} in (5)	π_1^{GP} in (16)
group	π_1^{GU} in (15)	π_1^{GP} in (16)	π^{GG} in (6)
	π_0^{GU} in (14)	π_0^{GP} in (16)	π^{GG} in (6)

We solve for the firms' equilibrium pricing schemes. We find that both firms employ personalized pricing in equilibrium.

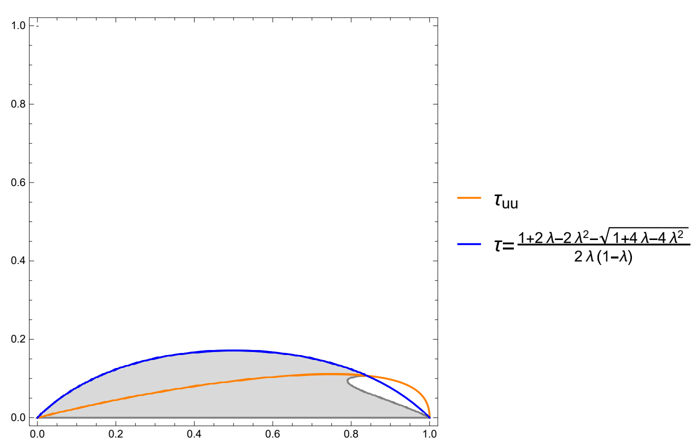
Proposition 3 *Both firms employ personalized pricing as a unique equilibrium outcome.*

The resulting outcome is expected given the outcome in Thisse and Vives (1988). If we pick up only personalized and group pricing, personalized pricing is the dominant strategy. Similarly, if we pick up only personalized and uniform pricing, personalized pricing is still the dominant strategy. Therefore, we find that personalized pricing is attainable in equilibrium. The result implies that the firms can be better off if they have coarser information about consumers, aligning with the finding in Laussel, Long, and Resende (2020).[8]

4.2 The effects of pricing schemes on profits and welfare

We numerically compare π^{UU} in (4), π^{PP} in (5), and π^{GG} in (6), due to the mathematical complexity.

Proposition 4 $\pi^{UU} < \pi^{PP} < \pi^{GG}$ *if and only if the parameter pair* (λ, τ) *is in the gray area in Figure 6; otherwise,* $\pi^{PP} < \pi^{UU} < \pi^{GG}$*. The border curve of the gray area consists of the pair* (λ, τ) *that satisfies* $\pi^{PP} = \pi^{UU}$.



Horizontal axis: λ; Vertical axis: τ.

Figure 6: The condition that $\pi^{UU} < \pi^{PP}$

When the mismatch costs of the two consumer groups are sufficiently heterogeneous, personalized pricing is better than uniform pricing. This contrasts with the standard result in the context of personalized pricing.

The intuition behind Proposition 4 is explained as follows. Because type L consumers incur a lower transportation cost compared with type H consumers, the former is more price-sensitive. When both firms adopt uniform pricing, they must offer sufficiently low prices to capture the price-elastic consumers, detrimental to the profit obtained from type H consumers. In contrast, adopting personalized pricing enables firms to delink the markets for the two types of consumers by offering customized prices to each individual. The competition mode becomes similar to an asymmetric Bertrand competition for each consumer. Then, personalized pricing brings each firm the following trade-offs: on the one hand, it intensifies the competition for type L consumers (a negative effect); on the other hand, each firm can set higher prices for some of type H consumers who located closer to it compared with the rival (a positive effect). The positive effect dominates when the heterogeneity between the two types is sufficiently large. In sum, under personalized pricing, there is a market segmentation with the outcome that both firms exploit less surplus from type L consumers but exploit more from some type H consumers who show strong brand preferences towards a particular firm (located geographically closer to one firm).

We discuss how the demand condition is important to evaluate the profitability of personalized pricing, comparing our result with Esteves (2022), who derives a reversal result in which personalized pricing is more profitable than uniform pricing, which is a reversal result to Thisse and Vives (1988). Esteves (2022) introduces two types of consumers in a Hotelling model: those who demand only one unit (type L consumers) and those who demand $q(> 1)$ units of the product (type H consumers). As q increases, type H consumers become more price elastic and more profitable for firms. The reversal result occurs only if the ratio of type H consumers is smaller than 50% and q is larger than 5. As q increases, the possibility of achieving the reversal result increases (the feasible ratio of type H becomes wider), with 20%

of type H consumers being highly desirable to achieve the reversal result.[9] The relationship between q (higher price elasticity) and the higher possibility of achieving the reversal result aligns with ours (see Figure 6). However, the preferable ratio of type H to achieve the reversal result (around 20%) differs from ours (around 50%) (see Figure 6) because of the double meaning of q in Esteves (2022): high price elasticity and high demand. The latter meaning distorts the ratio in Esteves (2022).

We investigate consumer welfare and numerically show that the flip side of Proposition 4 holds.

Proposition 5 *The consumer surplus in UU is the highest, and that in GG is the lowest if and only if the parameter pair (λ, τ) is in the gray area in Figure 6; otherwise, the consumer surplus in PP is the highest and that in GG is the lowest. The border curve of the gray area consists of the pair (λ, τ) that satisfies $\pi^{PP} = \pi^{UU}$.*

Because of the inelastic demand in the Hotelling model, the welfare ranking between the pricing schemes is opposite to the profit ranking.

Case GG is always the worst for consumer welfare among the three cases, UU, PP, and GG. Although UU can be the best for consumer welfare when the group heterogeneity is high, it is not easy to intervene in personalized pricing because such an intervention could facilitate group pricing, which is acceptable in practice, leading to the worst case.

Finally, we mention the total surplus in each case. In GG and PP, there is a unique pure strategy equilibria in which the firms equally split the market demand, achieving the first-best allocation. However, in UU, there is a mix strategy equilibrium if $\tau < \tau_{uu}$ because the *ex-post* outcome can be inefficient because of the *ex-post* asymmetric prices. Otherwise, a pure strategy equilibrium in UU is also the first-best.

Proposition 6 *PP and GG achieve the first-best allocation regardless of τ. If $\tau \geq \tau_{uu}$, UU also achieves the first-best allocation; otherwise, the ex-post outcome in UU can be* *inefficient.*

4.3 Additional pricing schemes

We discuss a case in which each firm has more pricing schemes than in the model we have considered. We add the following two pricing schemes: (i) uniform pricing for type H and personalized pricing for type L; (ii) uniform pricing for type L and personalized pricing for type H. The additions expand the payoff matrix from 3×3 to 5×5.

Firm 0/Firm 1	uni	personal	group	uni H personal L	uni L personal H
uni	✓	✓	✓	(∗∗) Firm 0 changes	(∗∗) Firm 0 changes
personal	✓	Equilibrium pair	✓	(∗) Firm 1 changes	(∗) Firm 1 changes
group	✓	✓	✓	(∗) Firm 1 changes	(∗) Firm 1 changes
uni H personal L	(∗∗) Firm 1 changes	(∗) Firm 0 changes	(∗) Firm 0 changes	(∗) Firm i changes	(∗) Firm i changes
uni L personal H	(∗∗) Firm 1 changes	(∗) Firm 0 changes	(∗) Firm 0 changes	(∗) Firm i changes	(∗) Firm i changes

We can show that the additional 16 pricing pairs are not achievable in equilibrium. First, without detailed calculations, we can show that the pairs of the pricing schemes with (∗) in the cells are not sustainable as an equilibrium. For those pricing pairs, the markets for the two types of consumers are delinked because the firms can offer different prices for the two types of consumers. The market segmentation implies that we can apply the result in Thisse and Vives (1998) in each market segment. Concretely, at least one of the firms has an incentive to change its pricing scheme to (complete) personalized pricing. Second, we can also show that the pairs of the pricing schemes with (∗∗) in the cells are not sustainable as an equilibrium because the firm that employs uniform pricing deviates to personalized pricing (the detail is available in the Online Appendix).

5 Conclusion

We examine firms' incentives to adopt one of the three pricing schemes: uniform, personalized, or group pricing in a Hotelling duopoly model. There are two types of consumer groups that are heterogeneous in their mismatch costs.

We show that regardless of the heterogeneity of consumer groups, both firms employ personalized pricing in equilibrium. Moreover, the profits when both firms employ personalized pricing are higher than those when they use uniform pricing if the heterogeneity of consumer groups is significant. The result is aligned with that of Esteves (2022). However, profits are always the highest among the three cases when the firms employ group pricing.

Additionally, if we focus on the three cases where both firms employ (i) uniform pricing (UU), (ii) personalized pricing (PP), and (iii) group pricing (GG), the consumer surplus in UU is the highest and that in GG is the lowest among the three cases if the heterogeneity of consumer groups is significantly high; otherwise, the consumer surplus in PP is the highest and that in GG is the lowest among the three cases. Therefore, group pricing is always the most harmful from the consumer welfare perspective. Furthermore, this result implies that it is not easy to intervene in personalized pricing because this could facilitate group pricing, leading to the worst case.

We can incorporate consumers' privacy concerns into our model by using the demand structure in Montes et al. (2019), where there are old and new consumers in a Hotelling duopoly model. In their model, a monopolistic data broker has the locational information of old consumers and can sell it to at least one of the firms. They consider each old consumer's incentive to delete the locational information from the database. As there are two dimensions of consumer characteristics in our model, location x and type t_k, each consumer has four options to manage the information characteristics. The richness of the consumer characteristics may lead to interesting results. Considering the extension can be future research.

Also, we can also consider firms' optimal degree of data collection. In our paper, we have assumed that firms have finer consumer information to implement personalized pricing. As in Laussel, Long, and Resende (2020), having coarser consumer information can be profitable for firms. Considering data collection in our model can be future research.

Notes

[1] OECD (2018) “Personalised pricing in the digital era,” 28 November 2018 https://one.oecd.org/document/DAF/COMP(2018)13/en/pdf (Last access: 13 March 2022).

[2] Bloomberg (2017) “Uber starts charging what it thinks you’re willing to pay,” 19 May 2017 https://www.bloombergquint.com/markets/uber-s-future-may-rely-on-predicting-how-much-you-re-willing-to-pay (Last access: 13 March 2022).

[3] Chen et al. (2020) examine a model wherein consumers’ identity management allows them to choose the uniform prices set only for new customers by firms. The model shows that personalized pricing can allow firms to fully extract the surpluses of targeted consumers.

[4] Lu and Matsushima (2023) discuss the effect of personalized pricing on profits and welfare in a Hotelling model in which consumers can simultaneously purchase from both firms (so-called multi-homing). They show the possibility that personalized pricing improves consumer welfare as well as firms’ profits. However, they do not consider heterogeneity in terms of mismatch costs.

[5] The Perloff and Salop (1985) model is also used in the context of mixed bundling (see Zhou, 2021).

[6] Wu (2021) mentions anecdotal evidence about price discrimination based on mobile phone types. https://kr-asia.com/researchers-took-over-800-trips-using-chinese-ride-hailing-apps-heres-what-they-found

[7] A survey conducted by Slickdeals, a US crowdsourced shopping platform, found that iPhone owners tend to have higher incomes and spend more compared to Android users. https://www.marketingdive.com/news/survey-iphone-owners-spend-more-have-higher-incomes-than-android-users/541008/

[8] By incorporating consumers’ identity management, Laussel, Long, and Resende (2022) further extend Laussel, Long, and Resende (2020).

[9]This ratio, 20%, aligns with the Pareto Principle in marketing (Twedt, 1964).

Appendix

The mixed strategy in Section 3.1 We can numerically obtain a symmetric mixed strategy equilibrium:[10]

$$p_{il}^{UUm} = \frac{(2\lambda\tau + \alpha^{UU}(1-\lambda)(1+\lambda\tau) + (\alpha^{UU})^2(1-\lambda)^2)t_H\tau}{2\lambda\tau(1-\lambda(1-\tau)) + \alpha^{UU}(1-\alpha^{UU})(1-\lambda)(1-\lambda-2\lambda\tau)},$$

$$p_{ih}^{UUm} = \frac{(\lambda(1+\tau-\lambda(1-\tau)) + \alpha^{UU}(1-\lambda)(1-\lambda(2-\tau)) - (\alpha^{UU})^2(1-\lambda)^2)t_H\tau}{2\lambda\tau(1-\lambda(1-\tau)) + \alpha^{UU}(1-\alpha^{UU})(1-\lambda)(1-\lambda-2\lambda\tau)},$$

$\alpha^{UU} = \beta^{UU} \in (0,1)$ that satisfy the following two equations:

$$\begin{aligned}
\pi_0^{UUm} &\equiv p_{0l}^{UUm}[\lambda\{\beta^{UU}x_H(p_{0l}^{UUm}, p_{1l}^{UUm}) + (1-\beta^{UU})x_H(p_{0l}^{UUm}, p_{1h}^{UUm})\} \\
&\qquad +(1-\lambda)\{\beta^{UU}x_L(p_{0l}^{UUm}, p_{1l}^{UUm}) + (1-\beta^{UU})\}] \\
&= p_{0h}^{UUm}[\lambda\{\beta^{UU}x_H(p_{0h}^{UUm}, p_{1l}^{UUm}) + (1-\beta^{UU})x_H(p_{0h}^{UUm}, p_{1h}^{UUm})\} \\
&\qquad +(1-\lambda)(1-\beta^{UU})x_L(p_{0h}^{UUm}, p_{1h}^{UUm})] \text{ and}
\end{aligned} \tag{A.1}$$

$$\begin{aligned}
\pi_1^{UUm} &\equiv p_{1l}^{UUm}[\lambda\{\alpha^{UU}(1-x_H(p_{0l}^{UUm}, p_{1l}^{UUm})) + (1-\alpha^{UU})(1-x_H(p_{0h}^{UUm}, p_{1l}^{UUm}))\} \\
&\qquad +(1-\lambda)\{\alpha^{UU}(1-x_L(p_{0l}^{UUm}, p_{1l}^{UUm})) + (1-\alpha^{UU})\}] \\
&= p_{1h}^{UUm}[\lambda\{\alpha^{UU}(1-x_H(p_{0h}^{UUm}, p_{1l}^{UUm})) + (1-\alpha^{UU})(1-x_H(p_{0h}^{UUm}, p_{1h}^{UUm}))\} \\
&\qquad +(1-\lambda)(1-\alpha^{UU})(1-x_L(p_{0h}^{UUm}, p_{1h}^{UUm}))].
\end{aligned}$$

Given the complexity of the last two simultaneous equations, we cannot explicitly derive the values of α^{UU} and β^{UU} but numerically show those values in Figure 2.

The mixed strategy in Section 3.5 We can obtain a mixed strategy NE:[11]

$$\begin{aligned}
p_0^m(H) &= \frac{t_H(9(1-\lambda)+4(1+2\lambda)\tau-\beta^{GU}(1-\lambda)(3-2\tau))}{3(3-(3-4\tau)\lambda+\beta^{GU}(1-\lambda))},\\
p_0^m(L) &= \frac{t_H\tau(12+\beta^{GU}-(\beta^{GU})^2-12\lambda+5\beta^{GU}\lambda+(\beta^{GU})^2\lambda+12\lambda\tau-6\beta^{GU}\lambda\tau)}{3(3-(3-4\tau)\lambda+\beta^{GU}(1-\lambda))},\\
p_{1l}^m &= \frac{t_H(6+5\beta^{GU}+(\beta^{GU})^2-6\lambda+7\beta^{GU}\lambda-(\beta^{GU})^2\lambda)\tau}{3(3-(3-4\tau)\lambda+\beta^{GU}(1-\lambda))},\\
p_{1h}^m &= \frac{t_H(9-9\lambda+2\tau+\beta^{GU}\tau+10\lambda\tau-\beta^{GU}\lambda\tau)}{3(3-(3-4\tau)\lambda+\beta^{GU}(1-\lambda))},
\end{aligned} \tag{A.2}$$

$\beta^{GU} \in (0,1)$ that satisfies the following equation:

$$\begin{aligned}
\pi_1^{GUm} &\equiv \lambda p_{1h}^m(1-x_H(p_0^m(H),p_{1h}^m))\\
&= p_{1l}^m[\lambda(1-x_H(p_0^m(H),p_{1l}^m))+(1-\lambda)(1-x_L(p_0^m(L),p_{1l}^m))].
\end{aligned}$$

Although we can explicitly derive the value of β which is complex, we numerically show this value in Figure 4.

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A Online Appendix

A.1 Both firms choose uniform pricing (UU)

We first show condition under which no firm unilaterally deviates from (p^{UU}, p^{UU}). By symmetry, assume that firm 0 deviates. Since firm 0 must obtain positive share, we exclude the case with $x_H = x_L = 0$. i.e., $p_0 \geq p^{UU} + t_H$. In addition, for any $p_0 \leq p^{UU} - t_H$, we have $x_H = x_L = 1$, which means that demand for firm 0 is constant. Then, for $p_0 \in [0, p^{UU} - t_H]$, firm 0's profit is maximized at $p_0 = p^{UU} - t_H$. Hence, it suffices to consider two types of deviation: (i) $p^{UU} - t_H \leq p_0 \leq p^{UU} - \tau t_H$ and (ii) $p^{UU} + \tau t_H \leq p_0 < p^{UU} + t_H$.

First, we consider case (i) in which $x_L = 1$. Given $p_1 = p^{UU}$, $\pi_0(p_0, p^{UU}) = \lambda p_0 x_H + (1 - \lambda)p_0$. The first-order condition yields the deviation price (with superscript $UD1$).

$$p_0^{UD1} = \frac{t_H[\lambda^2(1-\tau) - 3\lambda(1-\tau) + 2]}{2\lambda[1-\lambda(1-\tau)]}.$$

The necessary condition for this deviation is $p_0^{UD1} < p^{UU} - t_L$, which is always violated because

$$(p^{UU} - t_L) - p_0^{UD1} = -\frac{t_H[2\lambda^2\tau^2 + 3(1-\lambda)\lambda\tau + (2-\lambda)(1-\lambda)]}{2\lambda[1-\lambda(1-\tau)]} < 0.$$

Hence, firm 0 does not deviate in case (i).

Next, we consider case (ii) in which $x_L = 0$. With $p_1 = p^{UU}$, $\pi_0(p_0, p^{UU}) = \lambda p_0 x_H$. The first-order condition yields the deviation price (with superscript $UD2$).

$$p_0^{UD2} = \frac{t_H[1 + \tau - \lambda(1-\tau)]}{2 - 2\lambda(1-\tau)},$$

The deviation profit is

$$\pi_0^{UD2} = \frac{t_H\lambda[1 + \tau - \lambda(1-\tau)]^2}{8[1-\lambda(1-\tau)]^2}.$$

Comparing π^{UU} with π_0^{UD2} yields

$$\pi^{UU} - \pi_0^{UD2} = \frac{t_H(1-\lambda)[\lambda^2(1-\tau)^2 - \lambda(1+2\tau-3\tau^2)+4\tau]}{8[1-\lambda(1-\tau)]^2},$$

which is strictly positive if

$$\frac{2\sqrt{1-\lambda} - (2-\lambda-\lambda^2)}{\lambda(3+\lambda)} \equiv \tau^{UU} < \tau < 1.$$

Notice that under this condition, $p^{UU} + t_L - p_0^{UD2} \leq 0$ always holds. Hence, (p^{UU}, p^{UU}) is a pure strategy NE if $\tau > \tau^{UU}$.

For $0 < \tau \leq \tau^{UU}$, we derive the mixed NE as described in Equation (A.1). We solve four simultaneous equations derived from the following maximization system:

$$\max_{p_{0l}} \pi_{0l}^{UU} \equiv p_{0l}\lambda(\beta x_H(p_{0l}, p_{1l}) + (1-\beta)x_H(p_{0l}, p_{1h})) + p_{0l}(1-\lambda)(\beta x_L(p_{0l}, p_{1l}) + (1-\beta)),$$

$$\max_{p_{0h}} \pi_{0h}^{UU} \equiv p_{0h}\lambda(\beta x_H(p_{0h}, p_{1l}) + (1-\beta)x_H(p_{0h}, p_{1h})) + p_{0h}(1-\lambda)(1-\beta)x_L(p_{0h}, p_{1h}),$$

$$\max_{p_{1l}} \pi_{1l}^{UU} \equiv p_{1l}\lambda(\alpha(1-x_H(p_{0l}, p_{1l})) + (1-\alpha)(1-x_H(p_{0h}, p_{1l})) \\ + p_{1l}(1-\lambda)(\alpha(1-x_L(p_{0l}, p_{1l})) + (1-\alpha)),$$

$$\max_{p_{1h}} \pi_{1h}^{UU} \equiv p_{1h}\lambda(\alpha(1-x_H(p_{0h}, p_{1l})) + (1-\alpha)(1-x_H(p_{0h}, p_{1h}))) \\ + p_{1h}(1-\lambda)(1-\alpha)(1-x_L(p_{0h}, p_{1h})).$$

Next, we let $\pi_{0l}^{UU} = \pi_{0h}^{UU}$ and $\alpha = \beta$, and solve for α and β. The numerical outcome is in Figure 2.

A.2 One firm chooses uniform pricing and the other one chooses personalized pricing (UP)

We first consider the case with $0 \leq p_0 \leq \tau t_H$. The first-order condition yields

$$p_0^{UPl} \equiv \frac{t_H \tau}{2 - 2\lambda(1-\tau)}, \quad \pi_0^{UPl} \equiv \frac{t_H \tau}{8 - 8\lambda(1-\tau)}, \quad \pi_1^{UPl} \equiv \frac{t_H[9\tau + 4\lambda(1-\tau)^2 - 4\lambda^2(1-\tau)^2]}{16 - 16\lambda(1-\tau)},$$

where the superscript UPl denotes the case of $p_0^{UPl} \in [0, \tau t_H]$, which can be reduced to $0 < \lambda < 1/(2 - 2\tau) \equiv \lambda^{UPl}$ or $\tau \geq 1/2$.

Next, we consider the case with $\tau t_H \leq p_0 < t_H$. The first-order condition yields:

$$p_0^{UPh} \equiv \frac{t_H}{2}, \quad \pi_0^{UPh} \equiv \frac{t_H \lambda}{8}, \quad \pi_1^{UPh} \equiv \frac{t_H(8 + \lambda)}{16},$$

where the superscript UPh denotes the case of $p_0 \in [\tau t_H, t_H]$. Here, we derive the condition such that $\tau t_H \leq p_0^{UPh} \leq t_H$, from which we have $\tau \leq 1/2$.

From the above, we have two equilibrium candidates, p_0^{UPl} and p_0^{UPh}, if $0 < \lambda < \lambda^{UPl}$ and $0 < \tau < 1/2$, whereas there is a unique candidate, p_0^{UPh}, otherwise. Comparing π_0^{UPl} with π_0^{UPh} yields

$$\pi_0^{UPl} - \pi_0^{UPh} = \frac{t_H(1-\lambda)[\tau - \lambda(1-\tau)]}{8 - 8\lambda(1-\tau)} > 0 \Longleftrightarrow \lambda < \frac{\tau}{1-\tau} \equiv \lambda^{UP}.$$

Since $\lambda^{UPl} > \lambda^{UP}$ for $0 < \tau < 1/2$, the optimal p_0 is

$$p_0^{UP} = \begin{cases} \dfrac{t_H \tau}{2 - 2\lambda(1-\tau)} & \text{if} \quad 0 < \lambda \leq \dfrac{\tau}{1-\tau} \;\text{ or }\; 1/2 \leq \tau, \\ \dfrac{t_H}{2} & \text{if} \quad \dfrac{\tau}{1-\tau} < \lambda < 1 \;\text{ and }\; 0 < \tau < 1/2. \end{cases}$$

Using this result, we obtain the profit of firm 0.

$$\pi_0^{UP} = \begin{cases} \dfrac{t_H \tau}{8 - 8\lambda(1-\tau)} & \text{if} \quad 0 < \lambda \leq \dfrac{\tau}{1-\tau} \quad \text{or} \quad 1/2 \leq \tau, \\ \dfrac{t_H \lambda}{8} & \text{if} \quad \dfrac{\tau}{1-\tau} < \lambda < 1 \quad \text{and} \quad 0 < \tau < 1/2. \end{cases}$$

A.3 One firm chooses group pricing and the other one chooses uniform pricing (GU)

Due to symmetry, let firm 0 choose group pricing and offer $p_0(H)$ to market H and $p_0(L)$ to market L. Let firm 1 choose uniform pricing and offer p_1. The indifferent consumer in each market is given by $x_k^{GU}(p_0(k), p_1) = (t_k - p_0(k) + p_1)/(2t_k)$. Then, firm 0 and firm 1 solve

$$\max_{p_0(H), p_0(L)} p_0(H)\lambda x_H^{GU}(p_0(H), p_1) + p_0(L)(1-\lambda)x_L^{GU}(p_0(L), p_1),$$
$$\max_{p_1} p_1\{\lambda[1 - x_H^{GU}(p_0(H), p_1)] + (1-\lambda)[1 - x_L^{GU}(p_0(L), p_1)]\},$$

which yields the equilibrium prices $p_0^{GU*}(H)$, $p_0^{GU*}(L)$, p_1^{GU*} as in Eq. (10). Since firm 0's profit function with respect to $p_0(H)$ and $p_0(L)$ is globally concave, it has no incentive to deviate. However, given $p_0^{GU*}(H)$ and $p_0^{GU*}(L)$, firm 1's profit function is not globally concave, and may therefore unilaterally deviate from p_1^{GU*}. It can be confirmed that $\frac{t_H(1-\lambda)(1-\tau)}{2(1-\lambda+\lambda\tau))} < 0$, given $p_0^{GU*}(H)$, $p_0^{GU*}(L)$ and any $p_1 \in (0, 1)$, $x_H^{GU} > 0$. Moreover, firm 1 will never offer a price such that both $x_H = 1$ and $x_L = 1$, because its profit would become zero. Therefore, given $p_0^{GU*}(H)$, $p_0^{GU*}(L)$ firm 1's profit function is

$$\pi_1^{GU}(p_0^{GU*}(L), p_0^{GU*}(H), p_1)$$
$$\equiv \begin{cases} p_1\lambda[1 - x_H^{GU}(p_1)] & \text{if} \quad \frac{t_H\tau(4-3\lambda+3\lambda\tau)}{2(1-\lambda+\lambda\tau)} < p_1 \leq \frac{t_H(3-3\lambda+\tau+3\lambda\tau)}{2(1-\lambda+\lambda\tau)}, \\ p_1\{\lambda[1 - x_H^{GU}(p_1)] + (1-\lambda)[1 - x_L^{GU}(p_1)]\} & \text{if} \quad \frac{t_H\lambda(1-\tau)\tau}{2(1-\lambda+\lambda\tau))} \leq p_1 \leq \frac{t_H\tau(4-3\lambda+3\lambda\tau)}{2(1-\lambda+\lambda\tau)}, \\ p_1\{\lambda[1 - x_H^{GU}(p_1)] + (1-\lambda)\} & \text{if} \quad 0 < p_1 < \frac{t_H\lambda(1-\tau)\tau}{2(1-\lambda+\lambda\tau))}. \end{cases}$$

The second case induces the equilibrium price p_1^{GU}, so we discuss whether firm 1 deviates to remaining two cases: (i) $\frac{t_H\tau(4-3\lambda+3\lambda\tau)}{2(1-\lambda+\lambda\tau)} < p_1 \leq \frac{t_H(3-3\lambda+\tau+3\lambda\tau)}{2(1-\lambda+\lambda\tau)}$, and (ii) $0 < p_1 < \frac{t_H\lambda(1-\tau)\tau}{2(1-\lambda+\lambda\tau))}$.

First, we consider case (i) in which $x_L = 1$. Solving the first-order condition for p_1 yields the deviation price and profit (with superscript $GU1$):

$$p_1^{GU1} = \frac{t_H(3-3\lambda+\tau+3\lambda\tau)}{4(1-\lambda+\lambda\tau)},\ \pi_1^{GU1} = \frac{t_H\lambda(3-3\lambda+\tau+3\lambda\tau)^2}{32(1-\lambda+\lambda\tau)^2}$$

Since p_1^{GU1} always satisfies case (i), we compare π_1^{GU1} with π_1^{GU}, from which we have

$$\pi_1^{GU*} > \pi_1^{GU1} \iff \frac{4\sqrt{4-3\lambda}-(8-3\lambda-9\lambda^2)}{3\lambda(5+3\lambda)} \equiv \tau^{GU} < \tau < 1,$$

Next, we consider case (ii) in which $x_L = 0$. Solving the first-order condition for p_1 yields the deviation price (with superscript $GU2$)

$$p_1^{GU2} = \frac{t_H[(4-5\lambda+\lambda^2+5\lambda\tau-\lambda^2\tau)]}{4\lambda(1-\lambda+\lambda\tau)}.$$

It can be confirmed that p_1^{GU2} always violates case (ii). To summarize, $(p_0^{GU*}(L), p_0^{GU*}(H), p_1^{GU*})$ is a pure strategy NE if $\tau > \tau^{GU}$.

For $0 < \tau \leq \tau^{GU}$, we derive the mixed NE as described in Equation (A.2). We solve four simultaneous equations derived from the following maximization system:

$$\max_{p_0(H),p_0(L)} p_0(H)\lambda(\beta x_H(p_0(H), p_{1l}) + (1-\beta)x_H(p_0(H), p_{1h}))$$

$$+p_0(L)(1-\lambda)(\beta x_L(p_0(L), p_{1l}) + (1-\beta));$$

$$\max_{p_{1l}} \pi_{1l}^{GU} \equiv p_{1l}(\lambda(1-x_H(p_0(H), p_{1l})) + (1-\lambda)(1-x_L(p_0(L), p_{1l})));$$

$$\max_{p_{1h}} \pi_{1h}^{GU} \equiv p_{1h}(\lambda(1-x_H(p_0(H), p_{1h}))).$$

Next, we let $\pi_{1l}^{GU} = \pi_{1h}^{GU}$ and solve for β. The numerical outcome is Figure 4.

A.4 Equilibrium pricing schemes

We first show that GG, GP (PG), UP (PU), GU(GU) and UU cannot be an NE.

GG is not an NE: Given firm 0 choosing group pricing, firm 1's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_1^{GP} - \pi^{GG} = \frac{t_H(\tau + (1-\tau)\lambda)}{16} > 0.$$

GP (PG) is not an NE: Due to symmetry, we only consider GP. Given firm 1 choosing personalized pricing, firm 0's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_0^{PP} - \pi^{GP} = \frac{t_H(\tau + (1-\tau)\lambda}{8} > 0.$$

UP (PU) is not an NE: Due to symmetry, we only consider UP. From the results in UP (see Proof A.2), we have (i) UPl when $0 < \lambda \leq \tau/(1-\tau)$ or $\tau \geq 1/2$, and (ii) UPh when $\tau/(1-\tau) < \lambda < 1$ and $0 < \tau < 1/2$. In case (i), given firm 1 choosing personalized pricing, firm 0's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi^{PP} - \pi_0^{UPl} = \frac{t_H[2\lambda(1-\lambda)\tau^2 + (1-2\lambda)^2)\tau + 2\lambda(1-\lambda)]}{8(1-\lambda+\lambda\tau)},$$

which is always positive for any $\tau \in (0,1)$ and $\lambda \in (0,1)$. In case (ii), given firm 1 choosing personalized pricing, firm 0's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi_0^{PP} - \pi_0^{UPh} = \frac{t_H(\lambda + 2(1-\lambda)\tau)}{8} > 0.$$

UU is not an NE: when $\tau \geq \tau_{uu}$, we have pure strategy NE. We want to confirm that whether firm 1 would unilaterally deviate from choosing uniform pricing to choosing personalized pricing. From the results in UP (see Proof A.2), we have (i) UPl when $0 < \lambda \leq \tau/(1-\tau)$ or $\tau \geq 1/2$, and (ii) UPh when $\tau/(1-\tau) < \lambda < 1$ and $0 < \tau < 1/2$. In case (i), given firm 0 choosing uniform pricing, firm 1's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi_1^{UPl} - \pi^{UU} = \frac{t_H[4\lambda(1-\lambda)\tau^2 + (1-8\lambda+8\lambda^2)\tau + 4\lambda(1-\lambda)]}{16(1-\lambda+\lambda\tau)},$$

which is always positive for any $\tau \in (0,1)$ and $\lambda \in (0,1)$. In case (ii), given firm 0 choosing uniform pricing, firm 1's profit change from choosing uniform pricing to choosing personalized pricing is

$$\pi_1^{UPh} - \pi^{UU} = \frac{t_H[(8-7\lambda-\lambda^2) - (8-8\lambda-\lambda^2)\tau]}{16(1-\lambda+\lambda\tau)} > 0,$$

which is always positive for any $\tau \in (0,1)$ and $\lambda \in (0,1)$.

When $\tau < \tau_{uu}$, we have mixed strategy NE. Due to mathematical complexity involved, we can numerically show that each firm always has an incentive to deviate to choosing personalized pricing.

GU is not an NE: When $\tau \geq \tau_{gu}$, we have pure strategy NE. We want to confirm that whether firm 0 would unilaterally deviate from choosing group pricing to choosing personalized pricing. From the results in PU (see Proof A.2), we have (i) PUl when $0 < \lambda \leq \tau/(1-\tau)$ or $\tau \geq 1/2$, and (ii) PUh when $\tau/(1-\tau) < \lambda < 1$ and $0 < \tau < 1/2$. In case (i), given firm 1 choosing uniform pricing, firm 0's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_0^{PUl} - \pi_0^{GU} = \frac{t_H[2\lambda(1-\lambda)\tau^2 + (1-2\lambda)^2\tau + 2\lambda(1-\lambda)]}{16(1-\lambda+\lambda\tau)},$$

which is always positive for any $\tau \in (0,1)$ and $\lambda \in (0,1)$. In case (ii), given firm 1 choosing uniform pricing, firm 0's profit change from choosing group pricing to choosing personalized pricing is

$$\pi_0^{PUh} - \pi_0^{GU} = \frac{t_H[-2\lambda(1-\lambda)\tau^2 - (8-12\lambda+3\lambda^2)\tau + (8-9\lambda+\lambda^2)]}{16(1-\lambda+\lambda\tau)},$$

which is always positive for any $\tau \in (0,1)$ and $\lambda \in (0,1)$.

When $\tau < \tau_{gu}$, we have mixed strategy NE. Due to mathematical complexity involved, we can numerically show that each firm always has an incentive to deviate to choosing personalized pricing.

PP is an NE: We show that given firm 1 choosing personalized pricing, firm 0 does not deviate to choosing uniform pricing or to choosing group pricing. From the results in UP (see Proof A.2), we have (i) UPl when $0 < \lambda \leq \tau/(1-\tau)$ or $\tau \geq 1/2$, and (ii) UPh when $\tau/(1-\tau) < \lambda < 1$ and $0 < \tau < 1/2$. In case (i), firm 0's profit change from choosing personalized pricing to choosing uniform pricing is

$$\pi_0^{UPl} - \pi^{PP} = \frac{t_H(-2\lambda(1-\lambda)\tau^2 - (1-2\lambda)^2\tau - 2\lambda(1-\lambda)}{8} < 0.$$

In case (ii), firm 0's profit change from choosing personalized pricing to choosing uniform pricing is

$$\pi_0^{UPh} - \pi^{PP} = -\frac{t_H(\lambda + 2(1-\lambda)\tau}{8} < 0.$$

Moreover, firm 0's profit change from choosing personalized pricing to choosing group pricing is

$$\pi_0^{UPh} - \pi^{PP} = -\frac{t_H(\lambda + (1-\lambda)\tau}{8} < 0.$$

A.5 Proof for Subsection 4.3

Without a loss of generality, we consider the following two cases: (i) firm 0 chooses uniform pricing, whereas firm 1 chooses uniform pricing for type H and personalized pricing for type L (hUlP); (ii) firm 0 chooses uniform pricing, whereas firm 1 chooses uniform pricing for type L and personalized pricing for type H (hPlU).

(i) **hUlP:** Suppose $x_H(p_0, p_1(H)) \in (0,1)$ and $x_L(p_0, p_1(x,L)) \in (0,1)$. That is, $p_1(H) - t_H \leq p_0 < t_H\tau$. Firm 1's personalized prices offered to type L consumers are given by $p_1(x,L)) = p_0 + (2x-1)t_H\tau$. Then, firms solve the following maximization system:

$$\max_{p_0} \pi_0 = (x_H\lambda + x_L(1-\lambda))p_0,$$

$$\max_{p_1(H)} \pi_1 = (1-x_H)\lambda p_1(H) + (1-\lambda)\int_{x_L}^{1}(p_0 + (2x-1)t_H\tau)dx,$$

$$s.t.,\ p_1(H) - t_H \leq p_0 < t_H\tau,$$

from which we have the equilibrium prices:

$$\underline{p}_0^{hUlP} = \frac{t_H(2+\lambda)\tau}{4-4\lambda+3\lambda\tau}, \underline{p}_1^{hUlP}(H) = \frac{t_H(2-2\lambda+\tau+2\lambda\tau)}{4-4\lambda+3\lambda\tau},$$

and the corresponding equilibrium profits $\underline{\pi}_0^{hUlP}$, $\underline{\pi}_1^{hUlP}$. To derive the existence condition, it suffices to ensure that firm 0 does not deviate given $\underline{p}_1^{hUlP}(H)$. First, firm 0 never deviates by letting $x_L \geq 1$, because its profit increases in p_0 for $p_0 \leq \underline{p}_1^{hUlP}(H) - t_H$, and $p_0 = \underline{p}_1^{hUlP}(H) - t_H$ is dominated by $\underline{p}_0^{hUlP}$. Then, firm 0 can only deviate by letting $x_L \leq 0$ through offering a deviation price $\underline{p}_0^d$ such that

$$\underline{p}_0^d = \quad \max_{p_0} \underline{\pi}_0^d = \lambda x_H(p_0, \underline{p}_1^{hUlP}(H), p_1(x,L))p_0,$$

$$s.t.\ \underline{p}_0^d \in [t_H\tau, p_1^{hUlP}(H) + t_H).$$

from which we have

$$\underline{p}_0^d = \frac{t_H(6 - 6\lambda + \tau + 5\lambda\tau)}{2(4 - 4\lambda + 3\lambda\tau)},\ \underline{\pi}_0^d = \frac{t_H\lambda(6 - 6\lambda + \tau + 5\lambda\tau)^2}{8(4 - 4\lambda + 3\lambda\tau)^2}.$$

Such a deviation does not happen if $\underline{\pi}_0^d < \underline{\pi}_0^{hUIP}$, or $\underline{p}_0^d$ does not fall into the interval $[t_H\tau, \underline{p}_1^{hUIP}(H)+ t_H)$, from which we have the condition

$$\tau > \underline{\tau}^{hUIP} \equiv \frac{2(-4 - \lambda + 14\lambda^2) + 2\sqrt{16 + 8\lambda + 24\lambda^2 + 26\lambda^3 + 7\lambda^4}}{3\lambda(5 + 7\lambda)}.$$

Now, suppose $x_H(p_0, p_1(H), p_1(x, L)) \in (0, 1)$ and $x_L(p_0, p_1(H), p_1(x, L)) \leq 0$. That is, $t_H\tau \leq p_0 < p_1(H) + t_H$. Firm 1's personalized prices offered to type L consumers are given by $p_1(x, L)) = p_0 + (2x - 1)t_H\tau$. Then, firms solve the following maximization system:

$$\begin{aligned}
&\max_{p_0} \bar{\pi}_0 = x_H\lambda p_0,\\
&\max_{p_1(H)} \bar{\pi}_1 = (1 - x_H)\lambda p_1(H) + (1 - \lambda)\int_0^1 (p_0 + (2x - 1)t_H\tau)dx,\\
&s.t.,\ t_H\tau \leq p_0 < p_1(H) + t_H,
\end{aligned}$$

from which we have the equilibrium prices:

$$\bar{p}_0^{hUIP} = \bar{p}_1^{hUIP}(H) = t_H,$$

and the corresponding equilibrium profits $\bar{\pi}_0^{hUIP} = \frac{t_H\lambda}{2}, \bar{\pi}_1^{hUIP} = \frac{t_H(2-\lambda)}{2}$. To derive the existence condition, it suffices to ensure that firm 0 does not deviate given $\bar{p}_1^{hUIP}(H)$. First, firm 0 never deviates by letting $x_L \geq 1$, because its profit increases in p_0 for $p_0 \leq \bar{p}_1^{hUIP}(H) - t_H$, and $p_0 = \bar{p}_1^{hUIP}(H) - t_H$ is dominated by $\bar{p}_0^{hUIP}$. Then, firm 0 can only deviate by letting $x_L \in (0, 1)$

through offering a deviation price $\bar{p}_0^d$ such that

$$\bar{p}_0^d = \max_{p_0} \bar{\pi}_0^d = (\lambda x_H(p_0, p_1^{hUlP}(H), p_1(x,L)) + (1-\lambda)x_L(p_0, p_1(H), p_1(x,L)))p_0,$$
$$s.t.\ p_0 \in (\bar{p}_1^{hUlP}(H) - t_H, t_H\tau).$$

from which we have

$$\bar{p}_0^d = \frac{t_H(1+\lambda)\tau}{2(1-\lambda+\lambda\tau)},\ \bar{\pi}_0^d = \frac{t_H(1+\lambda)^2\tau}{8(1-\lambda+\lambda\tau)}.$$

Such a deviation does not happen if $\bar{\pi}_0^d < \bar{\pi}_0^{hUlP}$, or $\bar{p}_0^d$ does not fall in the interval $(\bar{p}_1^{hUlP}(H) - t_H, t_H\tau)$, from which we have the condition

$$\tau < \bar{\tau}^{hUlP} \equiv \frac{4\lambda}{1+3\lambda}.$$

To summarize, we have

$$\pi^{hUlP} = \begin{cases} \underline{\pi}_0^{hUlP}\ if\ \tau > \underline{\tau}^{hUlP}; \\ \bar{\pi}_0^{hUlP}\ if\ \tau < \bar{\tau}^{hUlP}, \end{cases}$$

where $\underline{\tau}^{hUlP} < \bar{\tau}^{hUlP}$ for any $\lambda \in (0,1)$.

Now, let us consider the case in which firm 0 deviates by choosing personalized pricing for both groups. Then,

$$x_H = \frac{t_H + p_1(H)}{2t_H},\ x_L = \frac{1}{2}.$$

Firm 1 solves the following maximization problem:

$$\hat{p}_1^{hUlP}(H) = \max_{p_1(H)} \pi_1 = \lambda(1-x_H)p_1(H) + (1-\lambda)\int_{1/2}^{1}((2x-1)t_H\tau)dx = \frac{t_H}{2},$$

from which we have $\hat{\pi}_0^{hUlP} = \frac{(9\lambda+4\tau-4\lambda\tau)t_H}{16}$.

Since $\hat{\pi}_0^{hUIP} > \pi_0^{hUIP}$, for firm 0 uniform pricing is dominated by personalized pricing.

(ii) **hPIU:** Suppose $x_H(p_0, p_1(x, H)) \in (0, 1)$ and $x_L(p_0, p_1(L)) \in (0, 1)$. That is, $p_1(L) - t_H\tau < p_0 < \min\{p_1(L) + t_H\tau, t_H\}$. Firm 1's personalized prices offered to type L consumers are given by $p_1(x, H)) = p_0 + (2x - 1)t_H$. Then, firms solve the following maximization system:

$$\max_{p_0} \pi_0 = (x_H\lambda + x_L(1 - \lambda))p_0,$$
$$\max_{p_1(L)} \pi_1 = +\lambda \int_{x_H}^{1} (p_0 + (2x - 1)t_H)dx + (1 - x_L)(1 - \lambda)p_1(L),$$
$$s.t., \ p_1(L) - t_H\tau < p_0 < \min\{p_1(L) + t_H\tau, t_H\},$$

from which we have the equilibrium prices:

$$p_0^{hPIU} = \frac{t_H(3 - \lambda)\tau}{3 - 3\lambda + 4\lambda\tau}, p_1^{hPIU}(L) = \frac{3t_H\tau - 2t_H\lambda\tau + 2t_H\lambda\tau^2}{3 - 3\lambda + 4\lambda\tau},$$

and the corresponding equilibrium profits $\pi_0^{hPIU}, \pi_1^{hPIU}$. With the equilibrium prices, $p_1^{hPIU}(L) + t_H\tau < (>) t_H$ if $\tau < (>) 1/2$.

First, let $\tau < 1/2$ such that $x_H(p_0, p_1(x, H)) \in (0, 1)$ and $x_L(p_0, p_1(L)) \in (0, 1)$ for $p_1^{hPIU}(L) - t_H\tau < p_0 < p_1^{hPIU}(L) + t_H\tau$. To derive the existence condition, it suffices to ensure that firm 0 does not deviate given $p_1^{hPIU}(L)$. Firm 0 never deviates by letting $x_L \geq 1$, because its profit increases in p_0 for $p_0 \leq p_1^{hPIU}(L) - t_H\tau$, and $p_0 = p_1^{hPIU}(L) - t_H\tau$ is dominated by p_0^{hPIU}. Then, firm 0 can only deviate by letting $x_L \leq 0$ through offering a deviation price p_0^d such that

$$p_0^d = \max_{p_0} \pi_0^d = \lambda x_H(p_0, p_1^{hPIU}(L), p_1(x, H))p_0, \ s.t. \ p_0 \in [p_1^{hPIU}(L) + t_H\tau, t_H),$$

from which we have

$$p_0^d = \frac{t_H}{2}, \ \pi_0^d = \frac{t_H\lambda}{8}.$$

Such a deviation does not happen if $\pi_0^d < \pi_0^{hPlU}$, or p_0^d does not fall into the interval $[p_1^{hPlU}(L)+t_H\tau, t_H)$, from which we have the condition

$$\tau > \tau^{hPlU} \equiv \frac{-9+6\lambda+5\lambda^2+\sqrt{81-108\lambda+27\lambda^2+6\lambda^3-2\lambda^4}}{6\lambda(3+\lambda)}.$$

For $\tau < \tau^{hPlU}$ we have a mixed-strategy NE in which firm 0 chooses a low price p_{0l}^{hPlU} with a probability γ such that $x_L \in (0,1)$ and a high price p_{0h}^{hPlU} such that $x_L \leq 0$. Then, firms solve the following maximization system:

$$\begin{aligned}
&\max_{p_{0l}} \pi_{0l} = p_{0l}(\lambda x_H(p_{0l}) + (1-\lambda)x_L(p_{0l}, p_1(L))),\\
&\max_{p_{0h}} \pi_{0h} = \lambda p_{0h} x_H(p_{0h}),\\
&\max_{p_1(L)} \pi_1 = \gamma\left[\lambda \int_{x_H(p_{0l})}^{1} (p_{0l}+(2x-1)t_H)dx + (1-\gamma)(1-\lambda)(1-x_L(p_{0l}, p_1(L)))p_1(L)\right]\\
&\quad +(1-\gamma)\left[\lambda \int_{x_H(p_{0h})}^{1} (p_{0h}+(2x-1)t_H)dx + (1-\gamma)(1-\lambda)p_1(L)\right],
\end{aligned}$$

from which we have

$$\begin{aligned}
p_{0l}^{hPlU} &= \frac{t_H(2+\gamma-2\lambda+\gamma\lambda)\tau}{\gamma(3-3\lambda+4\lambda\tau)},\\
p_{0h}^{hPlU} &= \frac{t_H}{2},\\
p_1^{hPlU}(L) &= \frac{t_H\tau(4-\gamma-4\lambda+2\gamma\lambda+4\lambda\tau-2\gamma\lambda\tau)}{\gamma(3-3\lambda+4\lambda\tau))},\\
\pi_{0l}(p_{0l}^{hPlU}) &= \frac{t_H(2+\gamma-2\lambda+\gamma\lambda)^2\tau(1-\lambda+\lambda\tau)}{2\gamma^2(3-3\lambda+4\lambda\tau)^2},\\
\pi_{0h}(p_{0h}^{hPlU}) &= \frac{t_H\lambda}{8}.
\end{aligned}$$

The probability γ^{hPlU} solves $\pi_{0l}(p_{0l}^{hPlU}) = \pi_{0h}(p_{0h}^{hPlU})$. Substituting γ^{hPlU}, we have

$$\pi_0^{hPlUm} = \gamma\pi_{0l}(p_{0l}^{hPlU}) + (1-\gamma)\pi_{0h}(p_{0h}^{hPlU}) = \frac{t_H\gamma}{8}.$$

From $0 < \gamma^{hPlU} < 1$, we have the existence condition for the mixed-strategy NE,

$$\tau < \tau^{hPlU}.$$

Now, let $\tau < 1/2$ such that $x_H(p_0, p_1(x, H)) \in (0, 1)$ and $x_L(p_0, p_1^{hPlU}(L)) \in (0, 1)$ for $p_1^{hPlU}(L) - t_H\tau < p_0 < t_H$. To derive the existence condition, it suffices to ensure that firm 0 does not deviate given $p_1^{hPlU}(L)$. Firm 0 never deviates by letting $x_L \geq 1$, because its profit increases in p_0 for $p_0 \leq p_1^{hPlU}(L) - t_H\tau$, and $p_0 = p_1^{hPlU}(L) - t_H\tau$ is dominated by p_0^{hPlU}. Then, firm 0 can only deviate by letting $x_L \leq 0$ through offering a deviation price p_0^{dd} such that

$$p_0^{dd} = \max_{p_0} \pi_0^{dd} = \lambda x_H(p_0, p_1(x, H))p_0, \ \ s.t. \ p_0 \in [t_H, p_1^{hPlU}(L) + t_H\tau),$$

from which we have

$$p_0^{dd} = \frac{t_H}{2}, \ \pi_0^{dd} = \frac{t_H\lambda}{8}.$$

Such a deviation does not happen if $\pi_0^{dd} < \pi_0^{hPlU}$, or p_0^{dd} does not fall into the interval $[t_H, p_1^{hPlU}(L) + t_H\tau)$, which is always true provided that $\tau > 1/2$.

Now, let us consider the case in which firm 0 deviates by choosing personalized pricing for both groups. Then,

$$x_H = \frac{1}{2}, \ x_L = \frac{t_H\tau + p_1(L)}{2t_H\tau}.$$

Firm 1 solves the following maximization problem:

$$\hat{p}_1^{hPlU}(L) = \max_{p_1(L)} \pi_1 = \lambda \int_{1/2}^{1} ((2x-1)t_H)dx + (1-\lambda)(1-x_L)p_1(L) = \frac{t_H\tau}{2},$$

from which we have $\tilde{\pi}_0^{hPlU} = \frac{t_H(4\lambda + 9\tau - 9\lambda\tau)}{16}$.

Since $\tilde{\pi}_0^{hPlU} > \pi_0^{hPlU}$ for $\tau > \tau^{hPlU}$ and $\tilde{\pi}_0^{hPlU} > \pi_0^{hPlUm}$ for $\tau < \tau^{hPlU}$, for firm 0 uniform pricing is dominated by personalized pricing.