Competitiveness and stability of collusive behavior

Supplemental material (not for publication)

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1 Another relative performance measure

We change the payoff of firm i $(i = 1, 2)$ as follows:

$$U_i = \pi_i + \alpha \frac{\pi_i}{\pi_j} \ (i \neq j),$$

where π_i is the profit of firm i and $\alpha \in (0, a^2/4)$. α indicates the importance of relative performance for firm i's management. Note that, when $\alpha = a^2/4$, the equilibrium price is zero in the competitive situation. We show it later.

First, we discuss joint-payoff maximization. The joint payoff is $\pi_1 + \pi_2 + \alpha\,(\pi_1/\pi_2 + \pi_2/\pi_1)$ and it is maximized when $y_1 = y_2 = a/2$. The resulting profit of each firm is $a^2/4$ (half of the monopoly profit), and the resulting payoff is:

$$U_1^C = \frac{a^2 + 8\alpha}{8},$$

where the superscript "C" denotes the outcome under the collusion.

Second, we discuss the deviation from the tacit collusion. Given the cooperative output of the rival, firm 2, firm 1 maximizes its payoff U_1. Given $y_2 = a/2$, the first-order condition is as below:

$$\frac{3a^2 + 16\alpha - 8ay_1}{4a} = 0.$$

From this, we obtain

$$y_1^D = \frac{3a^2 + 16\alpha}{8a},$$

where the superscript "D" denotes the outcome when a firm deviates from the collusion. The resulting payoff is:

$$U_1^D = \frac{(3a^2 + 16\alpha)^2}{8a}.$$

Third, we discuss the competitive situation. Each firm independently chooses its output so as to maximize its own payoff. We have the Cournot-Nash equilibrium as below:

$$y_1^E = y_2^E = \frac{a + \sqrt{a^2 + 12\alpha}}{6},$$

where the superscript "E" denotes the equilibrium outcome in the competitive phase. The resulting profit and payoff are given by:

$$\pi_1^E = \pi_2^E = \frac{(a + \sqrt{a^2 + 12\alpha})(2a - \sqrt{a^2 + 12\alpha})}{18}, \quad U_1^E = U_2^E = \frac{a^2 + 6\alpha + a\sqrt{a^2 + 12\alpha}}{18}.$$

Given y_1^E and y_2^E, the equilibrium price is

$$p^E = \frac{2a - \sqrt{a^2 + 12\alpha}}{3}.$$

This is zero when $\alpha = a^2/4$.

Results Given the collusive behavior of firm 2, firm 1 can increase its one-shot profit by deviating from the cartel. Its payoff is U_1^D. This deviation induces the competition thereafter. Firm 1's payoff at the competitive phase is U_1^E. If firm 1 does not deviate from the collusion, its current payoff is U_1^C. If firm 1 has no incentive for deviation now, it will have no incentive in future, as well. Thus, the tacit collusion is sustainable if and only if:

$$\frac{U_1^C}{(1-\delta)} \geq U_1^D + \frac{\delta U_1^E}{1-\delta}.$$

Let δ^* be the δ satisfying the above equation with equality. The tacit collusion is sustainable if and only if $\delta \geq \delta^*$. We have

$$\delta^* = \frac{U_1^D - U_1^C}{U_1^D - U_1^E} = \frac{9(a^2 + 16\alpha)^2}{(7a^2 + 48\alpha)^2 - 32a^3\sqrt{a^2 + 12\alpha}}.$$

Following the tradition of this field, we measure the stability of collusion in terms of this minimum discount factor δ^*. We have that an increase in α causes greater instability in collusive behavior.

Proposition *δ^* is increasing in α.*

Proof: The partial derivative of δ^* with respect to α is

$$\frac{\partial \delta^*}{\partial \alpha} = \frac{576a^2(a^2+16\alpha)[2(7a^2+48\alpha)\sqrt{a^2+12\alpha} - a(13a^2+144\alpha)]}{\sqrt{a^2+12\alpha}[(7a^2+48\alpha)^2 - 32a^3\sqrt{a^2+12\alpha}]^2}.$$

If the term between the brackets in the numerator is positive, $\partial\delta^*/\partial\alpha$ is also positive. The following difference has the same sign with this term:

$$[2(7a^2+48\alpha)\sqrt{a^2+12\alpha}]^2 - [a(13a^2+144\alpha)]^2.$$

The difference is $27(a^2+16\alpha)^3$. This is positive. $\partial\delta^*/\partial\alpha$ is positive. Q.E.D.

2 Price competition

We consider the case in which the firms compete in price. We set the demand system in this case as follows:

$$q_1 = \begin{cases} 0, & \text{if } a(1-\gamma)+\gamma p_2 \le p_1, \\ a - p_1, & \text{if } p_1 \le \dfrac{p_2 - a(1-\gamma)}{\gamma}, \\ \dfrac{a(1-\gamma) - p_1 + \gamma p_2}{1-\gamma^2}, & \text{otherwise,} \end{cases}$$

$$q_2 = \begin{cases} a - p_2, & \text{if } a(1-\gamma)+\gamma p_2 \le p_1, \\ 0, & \text{if } p_1 \le \dfrac{p_2 - a(1-\gamma)}{\gamma}, \\ \dfrac{a(1-\gamma) - p_2 + \gamma p_1}{1-\gamma^2}, & \text{otherwise.} \end{cases}$$

where γ is a positive constant ($\gamma \in (0,1)$). This demand system is related to standard demand functions for differentiated products. The payoff of firm i ($i = 1, 2$) is given by

$U_i = \pi_i - \alpha\pi_j$ $(i \neq j)$, where π_i is the profit of firm i and $\alpha \in (0,1)$. α indicates the importance of relative performance for firm i's management.

First, we discuss joint-payoff maximization. The joint payoff is $(1-\alpha)(\pi_1+\pi_2)$ and it is maximized when $p_1 = p_2 = a/2$. The resulting profit of each firm is $a^2/4(1+\gamma)$ (half of the monopoly profit), and the resulting payoff is:

$$U_1^C = \frac{(1-\alpha)a^2}{4(1+\gamma)},$$

where the superscript "C" denotes the outcome under the collusion.

Second, we discuss the deviation from the tacit collusion. Given the cooperative output of the rival, firm 2, firm 1 maximizes its payoff U_1. When $\alpha < (2-2\gamma-\gamma^2)/\gamma^2$, the optimal deviation price is an interior solution. Given $p_2 = a/2$, the first-order condition is as below:

$$\frac{(2-(1+\alpha)\gamma)a - 4p_1}{2(1-\gamma)(1+\gamma)} = 0.$$

From this, we obtain

$$p_1^D = \frac{(2-(1+\alpha)\gamma)a}{4},$$

where the superscript "D" denotes the outcome when a firm deviates from the collusion. The resulting payoff is:

$$U_1^D = \frac{(4(1-\alpha)(1-\gamma)+(1+\alpha)^2\gamma^2)a^2}{16(1-\gamma)(1+\gamma)}.$$

When $\alpha \geq (2-2\gamma-\gamma^2)/\gamma^2$, the optimal deviation price is a corner solution. This means that the demand for firm 2 is zero when firm 1 deviates from the collusion. Given $p_2 = a/2$, the optimal price is

$$p_1 = \frac{(2\gamma-1)a}{2\gamma}.$$

The resulting payoff is:

$$U_1^D = \frac{(2\gamma-1)a^2}{4\gamma^2}.$$

Third, we discuss the competitive situation. Each firm independently chooses its output so as to maximize its own payoff. We have the Bertrand-Nash equilibrium as below:

$$p_1^E = p_2^E = \frac{(1-\gamma)a}{2-(1-\alpha)\gamma},$$

where the superscript "E" denotes the equilibrium outcome in the competitive phase. The resulting profit and payoff are given by:

$$\pi_1^E = \pi_2^E = \frac{a^2(1-\gamma)(1+\alpha\gamma)}{(1+\gamma)(2-\gamma+\alpha\gamma)^2}, \quad U_1^E = U_2^E = \frac{a^2(1-\alpha)(1-\gamma)(1+\alpha\gamma)}{(1+\gamma)(2-\gamma+\alpha\gamma)^2}.$$

Results Given the collusive behavior of firm 2, firm 1 can increase its one-shot profit by deviating from the cartel. Its payoff is U_1^D. This deviation induces the competition thereafter. Firm 1's payoff at the competitive phase is U_1^E. If firm 1 does not deviate from the collusion, its current payoff is U_1^C. If firm 1 has no incentive for deviation now, it will have no incentive in future, as well. Thus, the tacit collusion is sustainable if and only if:

$$\frac{U_1^C}{(1-\delta)} \geq U_1^D + \frac{\delta U_1^E}{1-\delta}.$$

Let δ^* be the δ satisfying the above equation with equality. The tacit collusion is sustainable if and only if $\delta \geq \delta^*$. We have

$$\begin{aligned}
\delta^* &= \frac{U_1^D - U_1^C}{U_1^D - U_1^E} \\
&= \begin{cases} \dfrac{(2-(1-\alpha)\gamma)^2}{4(2-\alpha)-8(1-\alpha)\gamma+(1-\alpha)^2\gamma^2}, & \text{if } \alpha < (2-2\gamma-\gamma^2)/\gamma^2, \\ \dfrac{(2-(1-\alpha)\gamma)^2(\gamma^2\alpha-1+\gamma+\gamma^2)}{-4+4(2-\alpha)\gamma-(1-10\alpha+\alpha^2)\gamma^2+(1-\alpha)\gamma^3(2(1+\alpha)\gamma-3-5\alpha)}, & \\ & \text{if } \alpha \geq (2-2\gamma-\gamma^2)/\gamma^2. \end{cases}
\end{aligned}$$

Following the tradition of this field, we measure the stability of collusion in terms of this minimum discount factor δ^*. We have that an increase in α causes greater instability in collusive behavior.

Proposition *δ^* is increasing in α.*

Proof: The partial derivative of δ^* with respect to α is

$$\begin{aligned}
\frac{\partial \delta^*}{\partial \alpha} &= \frac{U_1^D - U_1^C}{U_1^D - U_1^E} \\
&= \begin{cases} \dfrac{4(1-\gamma)(2-(1-\alpha)\gamma)(2+(1-\alpha)\gamma)}{[4(2-\alpha)-8(1-\alpha)\gamma+(1-\alpha)^2\gamma^2]^2}, & \text{if } \alpha < (2-2\gamma-\gamma^2)/\gamma^2, \\ \dfrac{(1+\alpha)\gamma^4(2-(1-\alpha)\gamma)[2-5\gamma+3\gamma^2+2\gamma^3+2(5\gamma-3)\alpha+\gamma(-1+5\gamma-2\gamma^2)\alpha^2]}{[-4+4(2-\alpha)\gamma-(1-10\alpha+\alpha^2)\gamma^2+(1-\alpha)\gamma^3(2(1+\alpha)\gamma-3-5\alpha)]^2}, & \\ & \text{if } \alpha \geq (2-2\gamma-\gamma^2)/\gamma^2. \end{cases}
\end{aligned}$$

In the former case, we easily find that this is positive ($\gamma \in (0,1)$ and $\alpha(0,1)$). In the latter case, note that $(2-2\gamma-\gamma^2)/\gamma^2$ is larger than 1 if $\gamma \leq 3/5$. This means that the latter case appears only if $\gamma > 3/5$ because $\alpha \leq 1$. Now consider the term between the brackets in the numerator. We can easily show that $2-5\gamma+3\gamma^2+2\gamma^3$ and the coefficients of α and α^2 are positive for any $\gamma > 3/5$. This means that the term between the brackets is positive. Therefore, in the former and the latter cases, the partial derivative of δ^* is positive. Q.E.D.