Supplemental material (not for publication)

1 The case of a generalized objective function

We consider exactly the same model as in the main text, except that the public firm's objective function is now given by

$$W = \alpha\{(U - p_0x_0 - p_1x_1) + (\pi_0 + \pi_1) + (u_0 + u_1)\} + (1-\alpha)\pi_0, \tag{1}$$

In this specification, $\alpha \in [0, 1]$ captures the weight attached to social welfare: when $\alpha = 0$, the public firm only maximizes its own profit just like the private firm. Note that $W = \alpha U + (1-\alpha)\pi_0$, where U is a representative consumer's utility. Taking w_0 as given, the public firm's problem is defined as

$$\max_{x_0} \quad \alpha U + (1-\alpha)\pi_0, \quad s.t.\ (p_0 - w_0)x_0 \geq 0.$$

The private firm's problem remains the same.

Lemma S1 *In the output market equilibrium, the budget constraint for the public firm is binding if and only if*

$$\frac{2-\gamma-2w_0+\gamma w_1}{2-\gamma-\gamma^2 w_0+\gamma w_1} \leq \alpha \leq 1.$$

The output levels are given by

$$x_0 = \frac{2-\gamma-2w_0+\gamma w_1}{2-\gamma^2}, \quad x_1 = \frac{1-\gamma-w_1+\gamma w_0}{2-\gamma^2}.$$

Otherwise, it is not binding, and the output levels are given by

$$x_0 = \frac{2-\gamma-(2-\alpha)w_0+\gamma w_1}{4-2\alpha-\gamma^2}, \quad x_1 = \frac{2-\alpha-\gamma-(2-\alpha)w_1+(1-\alpha)\gamma w_0}{4-2\alpha-\gamma^2}.$$

PROOF: The Lagrangian for the public firm's problem is formulated as

$$\begin{aligned}\mathcal{L} &= \alpha\left(x_0 + x_1 - \frac{x_0^2 + 2\gamma x_0 x_1 + x_1^2}{2}\right) \\ &\qquad + (1-\alpha)(1 - x_0 - \gamma x_1 - w_0)x_0 + \lambda(1 - x_0 - \gamma x_1 - w_0)x_0.\end{aligned}$$

Taking w_0 as given, the first-order condition is given by

$$\frac{\partial \mathcal{L}}{\partial x_0} = 0 \Leftrightarrow \alpha(1 - x_0 - \gamma x_1) + (1-\alpha)(1 - 2x_0 - \gamma x_1 - w_0) + \lambda(1 - 2x_0 - \gamma x_1 - w_0) = 0.$$

If the constraint is slack, and there exists an interior solution, the optimal quantity for firm 0 is

$$x_0 = \frac{1 - \gamma x_1 - (1-\alpha)w_0}{2 - \alpha}. \tag{2}$$

If the constraint is binding, on the other hand, the public firm sets the quantity so that the resulting profit is zero. The constraint and the optimal quantity in this case are

$$1 - x_0 - \gamma x_1 - w_0 = 0, \quad x_0 = \frac{(1+\lambda)(1-\gamma x_1) - (1 - \alpha + \lambda)w_0}{2 - \alpha + 2\lambda}. \tag{3}$$

The problem for the private firm is, on the other hand, much simpler. The optimal quantity for the private firm is then given by

$$x_1 = \frac{1 - \gamma x_0 - w_1}{2}. \tag{4}$$

Given these results, we now obtain the output level for each firm. Solving the first-order conditions (3) and (4), we obtain

$$x_0 = \frac{2 - \gamma - 2w_0 + \gamma w_1}{2 - \gamma^2}, \tag{5}$$

$$x_1 = \frac{1 - \gamma - w_1 + \gamma w_0}{2 - \gamma^2}, \tag{6}$$

$$\lambda = \frac{(2 - \alpha\gamma^2)w_0 - (1-\alpha)(2 - \gamma + \gamma w_1)}{2 - \gamma - 2w_0 + \gamma w_1}, \tag{7}$$

$$x_0 = \frac{2 - \gamma - 2(1-\alpha)w_0 + \gamma w_1}{4 - 2\alpha - \gamma^2}, \tag{8}$$

$$x_1 = \frac{2 - \alpha - \gamma - (2-\alpha)w_1 + (1-\alpha)\gamma w_0}{4 - 2\alpha - \gamma^2}, \tag{9}$$

$$\lambda = 0. \tag{10}$$

Q.E.D.

When $\alpha < 1$, the budget constraint may not bind for the public firm, which gives rise to a discontinuity in the reaction function of union 1 in the bargaining stage. Pure-strategy equilibria may fail to hold because of this discontinuity.

To see this, suppose that no regulation is imposed on the public firm, and its wage is determined as a result of collective bargaining between the firm and the union. Because the budget constraint may not bind, we need to consider two distinct cases. First, the problem for union 0 is defined as

$$\max_{w_0} \quad w_0 x_0 = \begin{cases} \dfrac{w_0(2-\gamma-2w_0+\gamma w_1)}{2-\gamma^2}, & \text{if } \alpha \geq \dfrac{2-\gamma-2w_0+\gamma w_1}{2-\gamma-\gamma^2 w_0+\gamma w_1} \equiv \alpha(w_0, w_1), \\ \dfrac{w_0(2-\gamma-(2-\alpha)w_0+\gamma w_1)}{4-2\alpha-\gamma^2}, & \text{otherwise.} \end{cases}$$

Similarly, the problem for union 1 is defined as

$$\max_{w_1} \quad w_1 x_1 = \begin{cases} \dfrac{w_1(1-\gamma-w_1+\gamma w_0)}{2-\gamma^2}, & \text{if } \alpha(w_0, w_1) \leq \alpha, \\ \dfrac{w_1(2-\alpha-\gamma-(2-\alpha)w_1+(1-\alpha)\gamma w_0)}{4-2\alpha-\gamma^2}, & \text{otherwise.} \end{cases}$$

Solving the maximization problem for union 0, we have:

$$w_0 = \begin{cases} \dfrac{2-\gamma+\gamma w_1}{4} & \text{if } \dfrac{2}{4-\gamma^2} < \alpha, \\ \dfrac{(1-\alpha)(2-\gamma+\gamma w_1)}{2-\alpha\gamma^2} & \text{if } \dfrac{8-\gamma^2-\sqrt{32-16\gamma^2+\gamma^4}}{8} < \alpha \leq \dfrac{2}{4-\gamma^2}, \\ \dfrac{2-\gamma+\gamma w_1}{4(1-\alpha)} & \text{if } \alpha \leq \dfrac{8-\gamma^2-\sqrt{32-16\gamma^2+\gamma^4}}{8}. \end{cases} \tag{11}$$

Similarly, solving the maximization problem for union 1, we have:

$$w_1 = \begin{cases} \dfrac{1-\gamma+\gamma w_0}{2}, & \text{if } \alpha(w_0, w_1) \leq \alpha, \\ \dfrac{2-\alpha-\gamma+(1-\alpha)\gamma w_0}{2(2-\alpha)}, & \text{otherwise.} \end{cases}$$

To show that no pure strategy equilibrium exists, we discuss the case in which $(8-\gamma^2-\sqrt{32-16\gamma^2+\gamma^4})/8 < \alpha \leq 2/(4-\gamma^2)$. The reaction functions of the unions are depicted in Figure S1. In this case, the reaction function of union 0 is equivalent to $\alpha(w_0, w_1) = \alpha$, and anywhere right (left) of $w_0(w_1)$ in the figure is the region where $\alpha > \alpha(w_0, w_1)$ ($\alpha < \alpha(w_0, w_1)$) holds. The reaction functions of the unions overlap at the following two points:

$$\text{[Case 1]} \quad w_0 = \frac{(1-\alpha)(4-\gamma-\gamma^2)}{4-(1+\alpha)\gamma^2}, \; w_1 = \frac{(1-\alpha\gamma)(2-\gamma^2)}{4-(1+\alpha)\gamma^2}, \text{ if } \alpha(w_0, w_1) \leq \alpha,$$

$$\text{[Case 2]} \quad w_0 = \frac{(1-\alpha)((2-\alpha)(4-\gamma)-\gamma^2)}{(4-\alpha\gamma^2)(2-\alpha)-\gamma^2}, \; w_1 = \frac{(1-\alpha\gamma)(4-2\alpha-\gamma^2)}{(4-\alpha\gamma^2)(2-\alpha)-\gamma^2}, \text{ if } \alpha(w_0, w_1) \geq \alpha.$$

At each of these points, we end up with a corner solution, but there is also an interior solution which union 1 in general prefers more.

We show that in Case 1, union 1 indeed has an incentive to change its wage level. The maximization problem for union 1 is

$$\max_{w_1} \frac{w_1(2-\alpha-\gamma-(2-\alpha)w_1+(1-\alpha)\gamma w_0)}{4-2\alpha-\gamma^2},$$
$$s.t. \;\; \alpha(w_0, w_1) \geq \alpha, \;\; where \;\; w_0 = \frac{(1-\alpha)(4-\gamma-\gamma^2)}{4-(1+\alpha)\gamma^2}.$$

The optimal wage level of union 1 is

$$w_1 = \frac{(1-\alpha\gamma)(4(2-\alpha)-(3-\alpha)\gamma^2)}{2(2-\alpha)(4-(1+\alpha)\gamma^2)}$$

which satisfies the constraint $\alpha(w_0, w_1) > \alpha$. That is, w_1 is the interior optimum of the maximization problem. Therefore, the payoff of union 1 increases by deviating from the corner solution.

Similarly, in Case 2, union 1 again has an incentive to change its wage level. The maximization problem for union 1 is

$$\max_{w_1} \frac{w_1(1-\gamma-w_1+\gamma w_0)}{2-\gamma^2},$$
$$s.t. \;\; \alpha(w_0, w_1) \leq \alpha, \;\; where \;\; w_0 = \frac{(1-\alpha)((2-\alpha)(4-\gamma)-\gamma^2)}{(4-\alpha\gamma^2)(2-\alpha)-\gamma^2}.$$

The optimal wage level of union 1 is

$$w_1 = \frac{(1-\alpha\gamma)(4(2-\alpha)-(3-\alpha)\gamma^2)}{2(4(2-\alpha)-(1+2\alpha-\alpha^2)\gamma^2)}$$

which satisfies the constraint $\alpha(w_0, w_1) < \alpha$. That is, w_1 is the interior optimum of the maximization problem. As above, the payoff of union 1 increases by deviating from the corner solution.

The reaction functions of the unions are depicted in Figure S1.

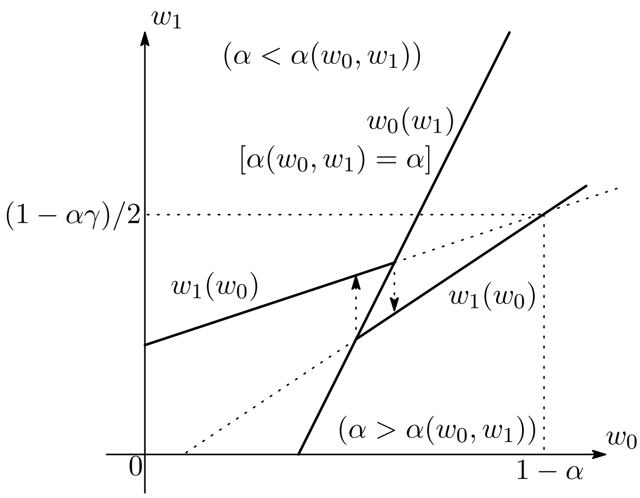

Figure S1: No pure strategy equilibrium exists.

¿From the discussion, we find that no pure strategy equilibrium exists if $(8-\gamma^2-\sqrt{32 - 16\gamma^2 + \gamma^4})/8 < \alpha \leq 2/(4 - \gamma^2)$.

2 The case of a monopoly union

2.1 The basic setup

We again consider the same model as in the main text, except that the unions are now integrated into an industry-wide union. Taking this as the reservation wage, the union sets the wages w_0 and w_1 to maximize the following utility function:

$$u_M = (w_0 - \bar{w})^{\theta} x_0 + (w_1 - \bar{w})^{\theta} x_1, \tag{12}$$

where θ is the weight the union attaches to the wage level. This setup implies that the union possesses full bargaining power. To focus our attention, we assume that $\theta = 1$ and $\bar{w} = 0$.

2.2 The unregulated public firm

We first consider a case where no regulation is imposed on the public firm, and its wage is determined as a result of collective bargaining between the firm and the union. Because of this, we need to consider the maximization problem. The problem for the monopoly union is defined as

$$\max_{w_0, w_1} \quad w_0 x_0 + w_1 x_1 = \frac{w_0(2 - \gamma - 2w_0 + \gamma w_1)}{2 - \gamma^2} + \frac{w_1(1 - \gamma - w_1 + \gamma w_0)}{2 - \gamma^2}.$$

Lemma S2 *Suppose that no regulation is imposed on the public firm. Then, the equilibrium wage and output levels, denoted as w_i^N and x_i^N, are given by*

$$w_0^N = w_1^N = \frac{1}{2}, \quad x_0^N = \frac{2 - \gamma}{2(2 - \gamma^2)}, \quad x_1^N = \frac{1 - \gamma}{2(2 - \gamma^2)}.$$

Proof: The first-order condition is given by

$$\frac{2 - \gamma - 4w_0 + 2\gamma w_1}{2 - \gamma^2} = 0, \quad \frac{1 - \gamma - 2w_1 + 2\gamma w_0}{2 - \gamma^2} = 0. \tag{13}$$

Solving and substituting them into the output levels derived in lemma 1 then yields the results.

Q.E.D.

In this case, the objective function of the union and social welfare are obtained, respectively, as

$$u_M^N = \frac{3-2\gamma}{4(2-\gamma^2)}, \quad W_M^N = \frac{19-14\gamma-8\gamma^2+6\gamma^3}{8(2-\gamma^2)^2}. \tag{14}$$

2.3 The regulated public firm

We now shift our attention to the case where the wage regulation is imposed on the public firm. Taking the equilibrium outcome in the output market as given, the union maximizes

$$u_M = w_0 x_0 + w_1 x_1 = \frac{kw_1(2-\gamma-2kw_1+\gamma w_1)}{2-\gamma^2} + \frac{w_1(1-\gamma-w_1+k\gamma w_1)}{2-\gamma^2}. \tag{15}$$

Lemma S3 *Suppose that the wage regulation is imposed on the public firm. Then, the equilibrium wage and output levels, denoted as w_i^R and x_i^R, are given by*

$$\begin{aligned} w_0^R &= \frac{k(1-\gamma+(2-\gamma)k)}{2(1-2\gamma k+2k^2)}, \quad w_1^R = \frac{1-\gamma+(2-\gamma)k}{2(1-2\gamma k+2k^2)}, \\ x_0^R &= \frac{4-\gamma-\gamma^2-(2+4\gamma-3\gamma^2)k+2(2-\gamma)k^2}{2(2-\gamma^2)(1-2\gamma k+2k^2)}, \\ x_1^R &= \frac{1-\gamma-(2+2\gamma-3\gamma^2)k+(4-2\gamma-\gamma^2)k^2}{2(2-\gamma^2)(1-2\gamma k+2k^2)}. \end{aligned}$$

Proof: Solving the problem for the union, we obtain the equilibrium wages. Substituting them into the output levels derived in lemma 1 then yields the equilibrium output levels.

Q.E.D.

In this case, the objective function of the union and social welfare are

$$u_M^R = \frac{(1-\gamma+(2-\gamma)k)^2}{4(1-2\gamma k+2k^2)(2-\gamma^2)}, \quad W_M^R = \frac{Z}{8(2-\gamma^2)^2(1-2\gamma k+2k^2)^2}, \tag{16}$$

where $Z \equiv 23-14\gamma-12\gamma^2+6\gamma^3+\gamma^4-2(6+40\gamma-33\gamma^2-19\gamma^3+13\gamma^4+\gamma^5)k+2(48-18\gamma+9\gamma^2-25\gamma^3-12\gamma^4+14\gamma^5)k^2-2(16+80\gamma-74\gamma^2-36\gamma^3+29\gamma^4+\gamma^5)k^3+(96-64\gamma-48\gamma^2+28\gamma^3+3\gamma^4)k^4$.

2.4 Comparison

Here, we will make the following two points. First, not surprisingly, workers are generally made better off under a monopoly union as they can now coordinate their wage demands. Second, this is generally welfare-reducing because higher equilibrium wages result in less output and higher equilibrium prices.

We first look at how union utility is affected when workers are allowed to form a monopoly union. When the unions are disintegrated, the unions' utilities under the regulation and those under no regulation case are

$$u_0^R = w_0^R x_0^R = \frac{k(1-\gamma)(4-\gamma-\gamma^2-2(1+\gamma-\gamma^2)k)}{4(1-k\gamma)^2(2-\gamma^2)},\ u_1^R = w_1^R x_1^R = \frac{(1-\gamma)^2}{4(1-k\gamma)(2-\gamma^2)}. \quad (17)$$

$$u_0^N = w_0^N x_0^N = \frac{2(4-\gamma-\gamma^2)^2}{(8-\gamma^2)^2(2-\gamma^2)},\ u_1^N = w_1^N x_1^N = \frac{(4-2\gamma-\gamma^2)^2}{(8-\gamma^2)^2(2-\gamma^2)}. \quad (18)$$

See the proof of Proposition 2 in the main text for details. We can then compute the difference in total union utility:

$$u_0^R + u_1^R - u_M^R = -\frac{(1-k)^2 k^2 (2-\gamma^2)}{4(1-2\gamma k+2k^2)(1-\gamma k)^2} \leq 0, \quad (19)$$

$$u_0^N + u_1^N - u_M^N = -\frac{\gamma^2(24-9\gamma^2-2\gamma^3)}{4(8-\gamma^2)^2(2-\gamma^2)} \leq 0. \quad (20)$$

In either case, one can see that workers are better off under a monopoly union.

As a flip side, the presence of a monopoly union is generally welfare-reducing. It follows from the main text (the proof of Proposition 5) that social welfare under the regulation case and that under no regulation case when the unions are disintegrated are

$$\begin{aligned} W^R &= \frac{23-4k^2-2(7+23k-4k^2)\gamma-(12-28k-23k^2)\gamma^2}{8(1-\gamma k)^2(2-\gamma^2)^2} \\ &\quad + \frac{2(3+12k-11k^2)\gamma^3+(1-12k-9k^2)\gamma^4-2k(1-4k)\gamma^5}{8(1-\gamma k)^2(2-\gamma^2)^2}, \quad (21) \\ W^N &= \frac{304-144\gamma-256\gamma^2+92\gamma^3+67\gamma^4-12\gamma^5-6\gamma^6}{2(8-\gamma^2)^2(2-\gamma^2)^2}. \quad (22) \end{aligned}$$

We can then compute the difference in social welfare:

$$W_0^R - W_M^R = \frac{(1-k)kY}{8(1-2\gamma k+2k^2)^2(1-\gamma k)^2(2-\gamma^2)} > 0, \quad (23)$$

$$W_0^N - W_M^N = \frac{\gamma((1-\gamma)(320+112\gamma-128\gamma^2-7\gamma^3)+\gamma^4(55-16\gamma-6\gamma^2))}{8(8-\gamma^2)^2(2-\gamma^2)^2} > 0, \quad (24)$$

where $Y \equiv 2(1-\gamma)(3-\gamma^2)+(1-\gamma)(2-22\gamma-\gamma^2+7\gamma^3)k+3(1-\gamma)(6-2\gamma+5\gamma^2+\gamma^3-2\gamma^4)k^2+(8-36\gamma+30\gamma^2+6\gamma^3-9\gamma^4+2\gamma^5)k^3+(8-16\gamma+6\gamma^2+4\gamma^3-3\gamma^4)k^4$. That is, social welfare generally decreases under a monopoly union, which might present a compelling case against it.